What is Linear?

Part of this activity is adapted from NCTM Navigation through Algebra in Grades 6 - 8.

Intended Audience Middle School Teachers of Algebra and Data Analysis

Goals: To model relationships which may or may not be linear and to increase ability to identify which relationships are linear and which are not.

Area vs. Perimeter

As a first example, let us consider a square of side length s. Recall that the perimeter of the square is $P = 4 \times \text{side} = 4s$. Likewise, the area is $A = \text{side} \times \text{side} = s^2$. We are now going to investigate how the perimeter and area of a square change as the side length changes.

Table Approach: Set up a table with 3 columns labeled *Side Length*, *Perimeter*, and *Area*, respectively. In the *Side Length* columns write the values 1, 1.5, 2, 1.5, ..., 9.5, 10. Compute the corresponding perimeters and areas and write those values in their respective columns.

Answer the following:

i) Based on your table which, if any, of these relationships appear to be linear in terms of side length? Justify your answer.

ii) If you were to graph perimeter and area vs. side length, what do you think the graphs would look like? Which graph do you think would increase at a faster rate?

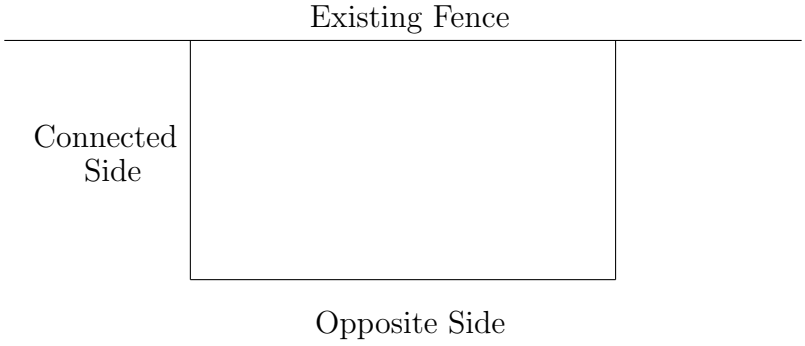
iii) Using either graph paper or your Graphing Calculator, make a graph of these two relationships. Use the x-axis for side length and the y-axis for both perimeter and area. Make sure to clearly identity the units for each of these three quantities.

iv) Your plot from *iii)* should consist of a discrete set of points (since your table contained a finite set of points). Does it make sense for this particular

application to draw a smooth curve through each set of points (i.e. one for perimeter and one for area) to make two continuous functions? Why or why not?

Combining Perimeter and Area

Now let us consider a slightly more sophisticated problem. Suppose that farmer Jack wants to enclose a rectangular plot of land. He has an existing fence which is quite long and can be used for one side of the enclosure. So he buys 32 feet of new fencing for the other three sides (see diagram below). Use tables, graphs, and diagrams (similar to the one below) to answer the following:



i) What is the largest area of land that can be enclosed by the fence?

ii) What is the length of the opposite side when the area is maximized? What is the length of the connecting side when the area is maximized?

iii) If you have not already done so, make plots of Area vs. Length of Connected Side and Length of Opposite Side vs. Length of Connected Side. Be sure to include units where necessary. Which, if any, of these relationships are linear? Justify your answer.

iv) Based on your graphs do you think there is a minimum area that can be achieved? Why or why not?

v) Write an equation that describes Area as a function of Length of Connected Side (you should already have a plot of points that are on the graph of this function). Graph this function on your graphing calculator. Use the maximization techniques described in your *An Introduction to Graphing Calculator Basics: Graphing Functions and Solving Equations* handout to find the maximum area. Does this agree with the answer you obtained previously?