Claim: Let $f: B \to B$ be a function where B is a finite set. Then f is 1-1 iff f is onto.

Proof:

Assume f is 1-1, we will show f is onto. Since B is finite we can list its elements: $b_1, b_2, \ldots b_n$. We can also list who f sends each element to:

$$\begin{aligned} * \quad f(b_1) &= b_{k_1} \\ f(b_2) &= b_{k_2} \\ f(b_3) &= b_{k_3} \\ &\vdots \\ f(b_n) &= b_{k_n} \end{aligned}$$

where $b_{k_i} \in B$ (note: there are n b_{k_i}) and $b_i \neq b_j$

Since f is 1-1 on a finite set with n elements we know each b_{k_i} is distinct; suppose not, then $b_{k_i} = b_{k_j}$ for some $i \neq j$, hence $f(b_i) = b_{k_i} = b_{k_j} = f(b_j)$ for $i \neq j$ which means $b_i = b_j$ for $i \neq j$ (since f is 1-1), which is a contradiction because each b_i is distinct. Now if each b_{k_i} is distinct f must be onto B because all the n b_{k_i} are listed as output.

Assume f is onto and consider the listing $*$ of what f does to B. Each element b_{k_i} is distinct (from above), is an element of B, and there are n of them. Hence $B = \{b_{k_1}, b_{k_2}, \ldots b_{k_n}\}$, f is onto B.