Mathematica exercise on linearization: K.M.Shannon 11/1/2015

Open a new notebook and complete the following:

A. Find linearizations for the following functions at the indicated points. Then determine the interval for which the linear approximation will be within .1 of the function.

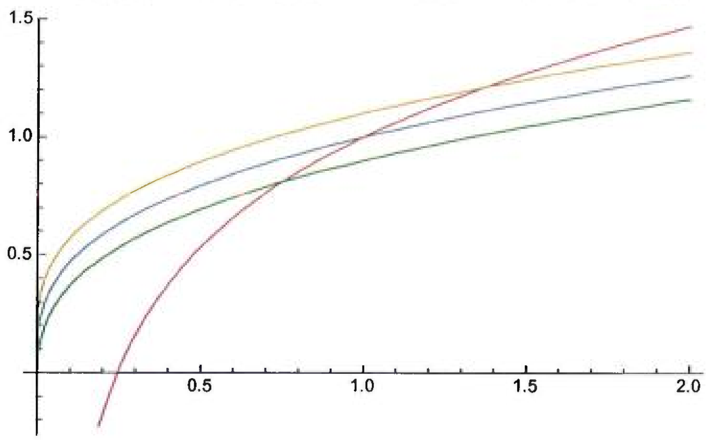
1. $\sqrt[3]{x}$ at x = 1
2. e^x at x = 0
3. ln(x) at x = 1

B. Use the linearizations you found above to approximate the following. Then have Mathemaitca compute them and calculate the "error".

1. $\sqrt[3]{0.9}$
2. $\sqrt[5]{e}$ (that is, $e^{0.2}$)
3. ln(1.1)

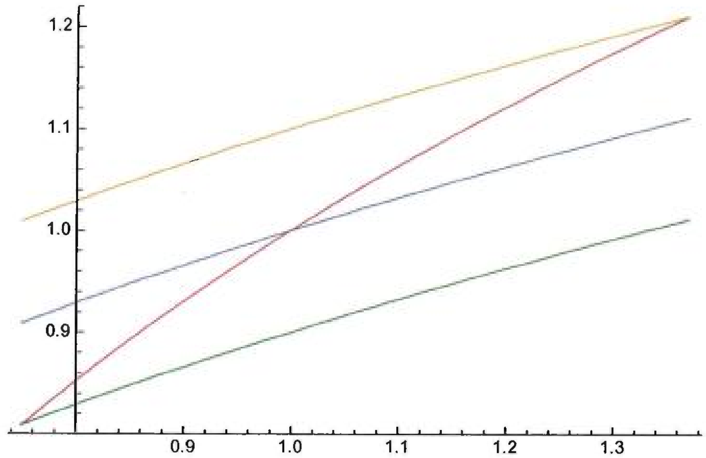
A 1. (note: I do not need to use SURD beacause x is positive.)

```
f[x_] := ∛x
L[x_] := f[x] + f'[x] (x - 1)
Plot[{f[x], L[x], f[x] + .1, f[x] - .1}, {x, 0, 2}]
```



It looks like the approximation should be good approximately .75 to 1.35. Let's take another look:

```
Plot[{f[x], L[x], f[x] + .1, f[x] - .1}, {x, 0.75, 1.37}]
```



Playing with the graph, it looks more like [.75, 1.37] Let's see if we can find it:

```
NSolve[L[x] == f[x] + .1, x]
```

{{x → 1.37007}}

```
NSolve[L[x] == f[x] - .1, x]
```

{{x → 0.751931}}

Make that [.751931,1.37007] but the eyeball estimate was pretty good! Now let's try B1:

```
L[.9]
```

0.929731

```
f[.9]
```

0.965489

```
f[.9] - L[.9]
```

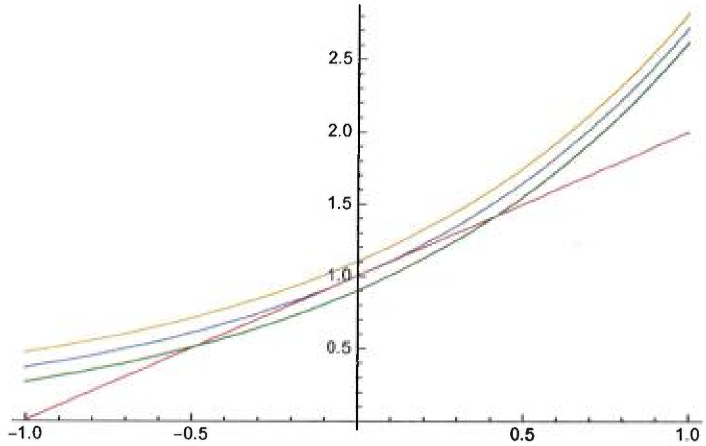
0.0357589

$\sqrt{0.9}$ = .965489 to six places. L[.9] is .929731 and the error in the approximation is .0357589. Now for A2:

```
f2[x_] := e^x
```

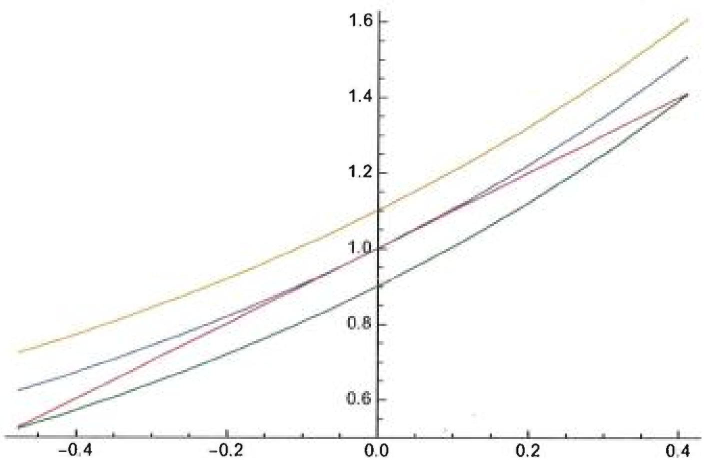
```
L2[x_] := f2[0] + f2'[0] x
```

```
Plot[{f2[x], L2[x], f2[x] + .1, f2[x] - .1}, {x, -1, 1}]
```



Looks like maybe -.47 to.47

```
Plot[{f2[x], L2[x], f2[x] + .1, f2[x] - .1}, {x, -.475, .41}]
```



or looking closer, maybe -.475 to .41. Lets try to find the solution:

```
NSolve[L2[x] == f2[x] - .1, x]
```

NSolve::ifun : Inverse functions are being used by NSolve,

so some solutions may not be found; use Reduce for complete solution information. >>

```
{{x → -0.483183}, {x → 0.416221}}
```

Well, I didn't think that was going to work. Let's try FindRoot....

```
FindRoot[L2[x] == f2[x] - .1, {x, .4}]
```

```
{x → 0.416221}
```

```
FindRoot[L2[x] == f2[x] - .1, {x, -.4}]
```

```
{x → -0.483183}
```

So the correct interval is [-.483183, .416221] to 6 places. Now lets do B2:

```
f2[.2]
```

1.2214

```
L2[.2]
```

1.2

```
f2[.2] - L2[.2]
```

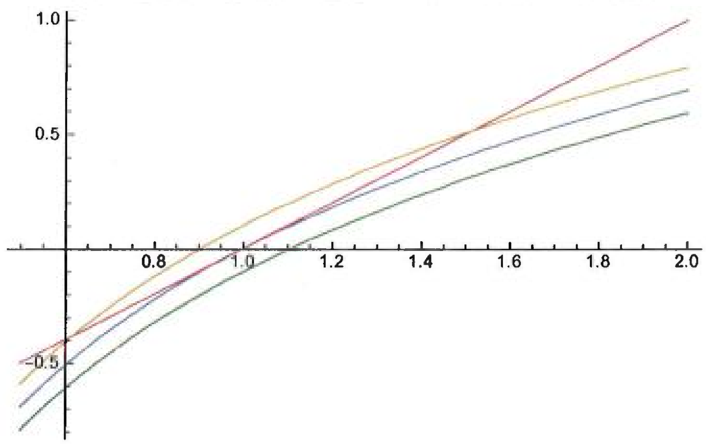
0.0214028

$\sqrt[5]{e}$ is1.2214 to 6 places, the linearization approximation is 1.2 and the error is .0214028. On to A3:

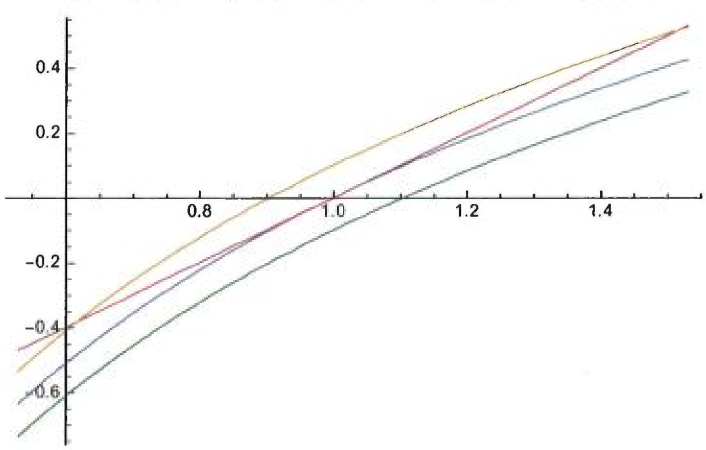
```
f3[x_] := Log[x]

L3[x_] := f3[1] + f3'[1] (x - 1)

Plot[{f3[x], L3[x], f3[x] + .1, f3[x] - .1}, {x, .5, 2}]
```




```
Plot[{f3[x], L3[x], f3[x] + .1, f3[x] - .1}, {x, .53, 1.53}]
```




```
FindRoot[L3[x] == f3[x] + .1, {x, 1.5}]
```

{x → 1.51622}

```
FindRoot[L3[x] == f3[x] + .1, {x, .5}]
```

{x → 0.616817}

Well the eyeball estimate wasn't as good this time. The interval we want is [.616817,1.51622]. Now for part B:

```
f3[1.1]
```

```
0.0953102
```

```
L3[1.1]
```

```
0.1
```

```
L3[1.1] - f3[1.1]
```

```
0.00468982
```

ln(1.1) = .0953102; the linear approximation is .1and the error is .00468982.
In all three examples, the linear approximation was pretty good, although the best in the natural log example.