Math 300 Test 1 Name Kms

1. If A = {a, b, c}, what is $\mathscr{P}$(A) = {{}, {a} {b} {c} {a,b} {a,c} {b,c} {a,b,c}}

2. For each of the following relations indicate for each if it is:
a)reflexive; b) symmetric c) antisymmetric d) transitive.
For each property that holds, explain why. For each property that does not hold, give a counter example.

i) The relation on the set of books in the library defined by $Book_a \sim Book_b$ if $Book_a$ and $Book_b$ have titles that begin with the same word.			
a)reflexive; yes a book has the same title as itself	b) symmetric Yes If $Book_a$'s title begins w/ the same word as $Book_b$'s then Visa Versa	c) antisymmetric No Discrete Mathematics ~ Discrete Dyn. Sys. and visa versa but they aren't the same	d) transitive yes A~B and B~C means they all start w/ the same so A~C
ii) The relation on the set A= {a, b, c} that consists of the following subset of AxA: {(a,a), (c,c)}			
a)reflexive; NO $b \not\sim b$	b) symmetric yes there are no cross rel. so a~a & a~a c~c and c~c.	c) antisymmetric yes the only way x~y and y~x is if x=y=a or x=y=c	d) transitive. yes there's no counter example.
iii) The relation on the set of student at SU defined by x~y if x has earned as many credits as y has.			
a)reflexive; yes Everyone has as many credits as themselves	b) symmetric No If A has 30 cr & B has 15 A~B but $B \not\sim A$	c) antisymmetric NO two new students would have 0 so A~B & B~A but $A \neq B$	d) transitive. yes if A has $\geq$ B and B has $\geq$ C then A~C.
iv) The relation on real numbers given by x~y $\lvert x-y \rvert < 1$			
a)reflexive; yes $\lvert x-x \rvert = 0 < 1$	b) symmetric yes $\lvert x-y \rvert = \lvert y-x \rvert$	c) antisymmetric NO 1.2 ~ 1.1 and 1.1 ~ 1.2 but $1.2 \neq 1.1$	d) transitive. NO 1 ~ 1.8 1.8 ~ 2.5 $1 \not\sim 2.5$
v) The relation on the set of natural numbers, N, defined by x~y if x and y are both odd.			
a)reflexive; NO $2 \not\sim 2$	b) symmetric yes if they both are odd that works both ways.	c) antisymmetric NO 3~5, 5~3 $3 \neq 5$	d) transitive. yes if x & y are odd as are y & z then so are x & z.
vi) The relation on $\mathscr{P}$(N) given by X~Y if $X \subseteq Y$.			
a)reflexive; yes any set is a subset of itself.	b) symmetric No $\{1\} \subseteq \{1,2\}$ $\{1,2\} \not\subseteq \{1\}$	c) antisymmetric yes, this is the defn of set equality!	d) transitive. yes if $A \subseteq B$ & $B \subseteq C$ clearly $A \subseteq C$
vii) The relation on A = {1, 2, 3, ..., 45} defined by x~y if x divides y (that is x|y).			
a)reflexive; yes everything divides itself	b) symmetric no $2 \mid 6$ $6 \nmid 2$	c) antisymmetric yes if a=nb and b=na a=n²a ⇒ n=±1 but there are no neg #'s in A.	d) transitive. yes. if a~b & b~c c=nb, b=ma ⇒ c=nma so a~c.

3. Let S be the set of students at SU and let C be the set of courses offered at SU. Define a relation between S and C. Student S is related to Course C if S took C and earned a grade of A.

4. Is it possible for a relation to be both antisymmetric and symmetric? Explain or give an example.
Yes. Any subset of equality is both symmetric and antisymetric since $x \sim y \Leftrightarrow x = y$

5. Why is it generally ok to disprove a universally quantified statement by a counterexample but not to prove a statement by example? Because the negation of a universally quantified statement is existentially quatified so all you need to show is $\exists$ a counter example but most general statements are statements about too many things to exhaustively show it is always true by checking every possible example.

6. Consider the statement: "All primes are odd."
a. What is its negation?
There is an even prime

b. What is it's converse?
all odds are prime

c. What is it's contrapositive?
Anything not odd is not prime
or No even numbers are prime.

7. State the axiom of induction. Why must it be an axiom rather than a theorem?
If $S \subseteq \mathbb{N}$ $\ni$:
1. $1 \in S$
2. whenever $k \in S$ then $k+1 \in S$
then $S = \mathbb{N}$

Complete 8, 9, and 10 on separate paper:
8. Prove that if x+y is even and y+z is even then x+z is even.

9. Chose two statements to prove by induction (prove the 3rd for Extra Credit):
a. 1 + 2 + ... + n = n(n+1)/2
b. if x≠1 then $\dfrac{x^n - 1}{x - 1} = \{x^{n-1} + x^{n-2} + \ldots + x + 1\}$
c. Every natural number can be written as the sum of distinct powers of 2.

10. Prove: If A B and B are sets then A∩(B∪C)' = A∩B'∩C' (use element chasing)