Math 300 Test 2 Name KMS

1. Let A = {♡, ♢, ♧, ♤, ☺}; B= {$x \in \mathbf{N} | 0 \le x \le 25$}; C = {1, 2, 3, 4, 5} For each of the following, either give an example or state why it is impossible to do so:

a. a function f: C->B which is one to one. $f(x) = x^2$ or $f(x) = x$

b. a function f: B->C which is one to one but not onto.

Not possible $|B| > |A|$

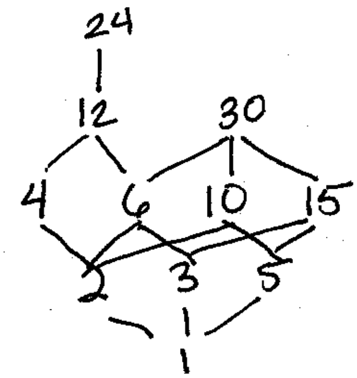
c. a function f: A->C which is one to one but not onto.

Not possible $|A| = |C| < \infty$ so any 1-1 function is a 1-1 corresp.

d. a function f: A->B which is onto.

Not possible $|A| < |B|$

5d)



2. Give an example of each of the following:

a. A function f:**N** -> **N** that is neither 1-1 nor onto.

$f(x) = 3$

b. A function f:**N** -> **N** that is 1-1 and onto.

$f(n) = n$

c. A function f:**N** -> **N** that is 1-1 but not onto.

$f(n) = n+1$ or $f(n) = n^2$

d. A function f:**N** -> **N** that is onto but not one to one.

$f(n) = |n-3|+1$ or $f(n) = \frac{n}{2}$ rounded.

3. Let A = {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}; B= {$x \in \mathbf{N} | -1 \le x \le 25$}, and f: A-> B be defined by $f(x) = x^2 - 1$.

a. What is $f^{-1}(\{15\})$? $\{-4, 4\}$

b. What is $f^{-1}(\{-1\})$? $\{0\}$

c. What is $f^{-1}(\{5\})$? $\emptyset$

d. What is $f^{-1}(\{0, 1, 2, 3, 4\})$? $\{-2, -1, 1, 2\}$

e. What is $f(\{-3, -1, 0, 1\})$? $\{0, -1, 8\}$

4. For each of the following relations, is it an equivalence relation, and if so describe the equivalence classes:

a. Given a function f:A -> B, define the relation ~ on A by x~y if f(x) = f(y)

yes, $\{x \in A \mid f(x) = c \in B\}$ one class for each element in $f(A) = Im(f) \subseteq B$

b. The relation on the set of students at SU defined by x~y if x is at least as tall as y.

no, not sym.

c. The relation on the set of integers defined by x~y if x-y is divisible by 3.

yes, $T_1 = \{\ldots -5, -2, 1, 4, 7, \ldots\}$ $T_2 = \{-4, -1, 2, 5, 8 \ldots\}$ $T_3 = \{\ldots -3, 0, 3, 6 \ldots\}$

5. Let A = {1, 2, 3,4, ..., 60} and consider the relation ~ where x~y if x divides y (that is x|y).

a. How many different sets are in Ω~, the collection of subsets associated with ~? List three of them, if possible, list three of different sizes. 60. $T_1 = A$, $T_{10} = \{10, 20, 30, 40, 50, 60\}$, $T_{60} = \{60\}$

4pts

b. Consider the subsets of A listed in the table below. For each answer the following questions with respect to the ordering ~: (the first set is an example for you)

i. Does the set have a lower bound? If so, list one.
ii. Does the set have a greatest lower bound? If so what?
iii. Does the set have an upper bound? If so, list one.
iv. Does the set have a least upper bound? If so what?
v. Does the set have a greatest element? If so what?
vi. Does the set have a least element? If so what?

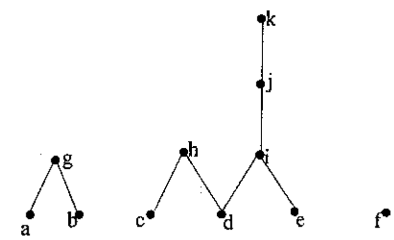
30 × 1/2 = 15

Set	i. l.b.	ii. g.l.b.	iii. u.b.	iv. l.u.b.	v. g.e.	vi. l.e.
{20, 30}	Yes, 5	yes, 10	yes, 60	yes, 60	no	no
all of the divisors of 60 (including 1)	Yes, 1	yes, 1	yes, 60	yes, 60	yes 60	yes 1
all of the primes in A	yes, 1	yes, 1	No	No	No	No
{2, 4, 8, 16}	yes 1	yes 2	yes 32	yes 16	yes 16	yes 2
{ }	Yes, 5	No ~~yes, 60~~	yes 5	yes 1	No	No
{2, 6, 28}	yes, 1	yes, 2	No	No	No	yes, 2

d. Consider the same ordering (that is x~y if x|y) on the set {1, 2, 4, 3, 5,6, 10, 12, 15, 24, 30}. On separate paper, draw a lattice diagram for this partially ordered set. See pg 1.

4 pts

6. Given the set A = {a, b, c, d, e, f, g, h, i, j, k} and the ordering shown in the lattice diagram to the right:



a. give an example of an unbounded subset of A. {g, h}

b. give an example of a bounded (both above and below) subset of A. {i, j}

c. which elements are maximal?
g, h, k, f

d. Is any element greatest? If so, which?
NO.

3072 = 3(900) + 372
900 = 2(372) + 156
372 = 2(156) + 60
156 = 2(60) + 36
60 = 36 + 24
36 = 24 + 12
24 = 2(12) + 0

7. Use the Euclidean Algorithm to find gcd(900, 3072). Show all work on separate paper!
4pts gcd = 12

8. List a pair of natural numbers that are not prime but are relatively prime 4, 9. List a pair that are neither prime nor relatively prime 25, 75.

9. Let A be the set of lower case letters in the standard alphabet and consider the following subsets Ω of 𝒫(A). In each case is ~Ω an equivalence relation? Why or why not? If it is not, state a property that fails:

a. {{a, e, i, o, u}, {a, b, c, d, e, f}, { g, k, p, q, t , y}, {f, h, j, l, r, s, v, w, x, z}, {m, n}}
No, not ~~sym~~ transitive b~a, a~u b≁u

b. Ω = {{a, b, c}, {d, e, f}, {x, y, z}}
No, not reflexive g≁g

c. ~~Ω~~ Ω = {{a, e, i, o, u}, {b, c, d, g, k, p, q, t , y}, {f, h, j, l, r, s, v, w, x, z}, {m, n}}
yes, Ω partitions A.