DE In Class exercises 9-30 see inclassx9-30.pdf for problems

A

1. $r^2-2r-3=0$ $(r-3)(r+1)=0$ $y(x)=c_1e^{3t}+c_2e^{-t}$

2. $r^2+10r+25=0$ $(r+5)^2=0$ $y(x)=e^{-5x}+xe^{-5x}$

3. $r^2+100=0$ $r=\pm 10i$ $y(x)=c_1\cos 10t+c_2\sin 10t$.

pg 230 $c_1\cos\omega t+c_2\sin\omega t = A\sin(\omega t+\phi)$ where $A=\sqrt{c_1^2+c_2^2}$, $\tan\phi = c_1/c_2$

B. $x_0=-3$ $x_0'=4$ $x''+x=0$ $x(t)=4\sin t-3\cos t = A\sin(t+\phi)$

$A=\sqrt{16+9}=5$ $\phi=\arctan(-3/4)$ Amplitude $=5$

1) $x(t)=c_1\cos t+c_2\sin t$ $x(0)=c_1=-3$

$x'(t)=-c_1\sin t+c_2\cos t$ $x'(0)=c_2=4$

2. $r^2+br+1/2=0$ if $b<\sqrt{2}$ we have oscillations

$b^2-4ac=b^2-2$ (Note $b>0$ so we don't need $-\sqrt{2}<b<\sqrt{2}$)

4. $e^{-t}\sin 2t$ $\alpha=-1$ $\beta=2$

$\alpha=\frac{-b}{2a}\Rightarrow -b=-2$ if $\beta=\frac{\sqrt{b^2-4ac}}{2a}$

let $a=1$, $b=2$ $\beta=\frac{\sqrt{4c-4}}{2}=\sqrt{c-1}$

3. The DE is just the sameas A2

$x(t)=e^{-5t}+te^{-5t}$

3. $r=-5$ double root $(r+5)^2\Rightarrow$ $\boxed{x''+10x'+25x=0}$ $4=c-1$ $c=5$

2. $\boxed{x''+100x=0}$

4. cont $x''+2x'+5=0$

$x''-2x'-3x=0$

C. 1. $r=3,-1$ $(r-3)(r+1)=r^2-2r-3$

5. need $r=1$ to be a triple root $(r-1)^3=r^3-3r^2+3r-1$: $\boxed{x'''-3x''+3x'-x=0}$

D. 1. $m=75$ $T_E=200$ $T_K=.9(200)+.1(75)=187.5$ $C_E=125$ $C_K=112.5$

$\frac{dT(t)}{dt}=k(75-T)$ $\frac{dT}{75-T}=k\,dt\Rightarrow \ln(T-75)=-kt+C\Rightarrow T-75=Ce^{-kt}$ $T=75+Ce^{-kt}$

$\frac{-75}{11\ 2.5}$

$T_E(10)=(75+125e^{-10k})\cdot 9+.1(75)=75+112e^{-10k}$

$T_K(10)=75+112e^{-10k}$ The two coffees are the same temp

if you redo w/ α & $1-\alpha$ instead of .9 & .1, you get the same result.

D2. $50\times.03$ → $A_0=380{,}000\times.05$ → $A_0=19000$; $25\times\frac{A}{V}$ → 1.5

$\frac{dA}{dt}=1.5-\frac{25A}{380{,}000+25t}$

$A'+\frac{25}{V_0+25t}A=1.5$ $\mu=e^{\int\frac{25}{V_0+25t}dt}=e^{\ln(V_0+25t)}=V_0+25t$

$A(0)=\frac{C}{V_0}\Rightarrow C=(19{,}000)(380{,}000)$

$(V_0+25t)A'+25A=1.5(V_0+25t)$

$V_0=380{,}000$

$(V_0+25t)A=1.5V_0t+\frac{75}{4}t^2+C$

Tank is filled when $t=\frac{20{,}000}{25}$

$A=\frac{1.5V_0t+18.75t^2+C}{V_0+25t}=\frac{1.5V_0t+18.75t^2+19000V_0}{V_0+25t}$

$t=800$ hours

Concentration $C(t)=\frac{1.5V_0t+18.75t^2+19000V_0}{(V_0+25t)^2}$

$C(800)=\frac{1.5V_0(800)+18.75(800)^2+19000V_0}{(400{,}000)^2}=.048$

Coming Soon... 4.6% question