Differential Equations In-class exercise:

Part A:

1. Find the general solution to $y''(x) - 2y'(x) - 3y(x) = 0$
2. Find the General Solution to $y''(x) + 10y'(x) + 25y(x) = 0$
3. Find the General Solution to $y''(x) + 100y(x) = 0$

Part B:(assume all units are consistent and can therefore be ignored)
The accepted differential model for motion of an object on a spring is:

$$mx'' + bx' + kx = f(t)$$

where x(t) is the distance the mass is from the equilibrium point; f(t) is an exterior forcing function, m is the mass of the object, b is the damping constant which depends on the materials involved and k is the spring constant. Suppose a mass of 1 unit is attached to a spring with spring constant k.

1) Suppose that k=1 and the spring is in a vacuum so that b=0, there is no forcing function, and the mass is pulled down 3 units and given an initial upward velocity of 4 units/sec. What is the amplitude of the resulting simple harmonic motion?

2) If k = 1/2, what values of b would result in oscillations?

3) Suppose k=25, the spring is immersed in a medium which causes the damping constant to be b=10, and there is no forcing function. Find the general solution to the differential equation (above) that governs it's motion.

Part C: Construct undriven, linear differential equations with constant coefficients for which the following are solutions:

1. $2e^{3t}+e^{-t}$ 2. $\cos(10t)$ 3. te^{-5t} 4. $e^{-t}\sin(2t)$ 5. t^2e^t

Part C: Complete the following Modeling problems:

1. Dr.'s Nagle & Saff visited SSU on a day when miraculously the Air conditioning system in Power was keeping the room temperature at a constant 75°. In the seminar room of the Power building, they each poured themselves a cup of piping hot 200° coffee at precisely the same time. Kent Nagle immediately added cream (at room temperature) to his coffee. They took the quick 10 minute tour of the Mathematics and Computer Science Department and returned to the seminar room where Ed Saff finally added room temperature cream to his coffee. Use Newton's Law of Cooling to model this situation and predict whose coffee is hotter. Would your answer change if the cream was kept in the refrigerator at 40° until used? How so? Discuss the strengths and weaknesses of this model.

2. When adding water to fill a fish tank whose water level is low, you are supposed to remove an amount equal to that by which it is low then add twice that amount of fresh water. Suppose the new manager of Sea World notices that Shamu's 400,000 ton tank is 20,000 tons low. (ie. there's only 380,000 tons of water in the tank) Also suppose that she decides to fill the tank by adding 50 tons per hour of a 3% salt water solution and bleeding off 25 tons per hour of the water in the tank. She will stop the process when the tank is filled. If the concentration of salt in the tank is initially 5% model the salt in the tank. Assume that Shamu keeps the tank well stirred. What is the concentration of salt at time t? When the tank is filled? What concentration of saline should she add if she desires a final concentration of 4.6%? Discuss the strengths and weaknesses of your model.