Math 311 Optional Test 1.5 Name KMS

1. Solve the following initial value problems:

a. $\frac{dy}{dz} = 5 - \frac{y}{z}$; y(1)=3.

$e^{\ln(z)} = z$

$y' + \frac{1}{z}y = 5$

$zy' + y = 5z$

$[zy]' = 5z$

$zy = \frac{5}{2}z^2 + C$

$y = \frac{5}{2}z + \frac{C}{z}$

$y(1) = \frac{5}{2} + C = 3 = \frac{6}{2} \Rightarrow C = \frac{1}{2}$

$\boxed{y(z) = \frac{5}{2}z + \frac{1}{2z}}$

b. $y' = (y^2/x^2) + (y/x)$; y(1) = 1

Let $v = \frac{y}{x}$ $\quad y = xv$ $\quad y' = v + xv'$

$v + xv' = v^2 + v$ $\quad xv' = v^2$ $\quad \frac{1}{v^2}v' = \frac{1}{x}$ $\quad -\frac{1}{v} = \ln(x) + C$ $\quad v = \frac{-1}{\ln(x)+C}$

$\frac{y}{x} = -\frac{1}{\ln(x)+C}$ $\quad y = -\frac{x}{\ln(x)+C}$

$y(1) = -\frac{1}{C} = 1 \Rightarrow C = -1$

$\boxed{y = \frac{x}{1-\ln(x)}}$

c. y'' - y = 2 e^t ; y(0)=0; y'(0)=3.

$r^2 - 1 = 0$

$y_h = c_1e^t + c_2e^{-t}$

$y_p = Ate^t$

$y_p' = Ae^t + Ate^t$

$y_p'' = 2Ae^t + Ate^t$

$2Ae^t + Ate^t - Ate^t = 2e^t$

$A = 1$

$y_g = c_1e^t + c_2e^{-t} + te^t$

$y_g(0) = c_1 + c_2 = 0$

$y_g' = c_1e^t - c_2e^{-t} + e^t + te^t$

$y_g'(0) = c_1 - c_2 + 1 = 3$

$c_1 - c_2 = 2$

$2c_1 = 2 \Rightarrow c_1 = 1$

$c_2 = -1$

$\boxed{y = e^t - e^{-t} + te^t}$

2. Find general solutions to the following:

a. y'' + 4y' +3y = 16 e^t

$r^2 + 4r + 3 = 0$

$(r+3)(r+1)$

$y_h = c_1e^{-3t} + c_2e^{-t}$

$y_p = Ae^t = y_p' = y_p''$

$Ae^t + 4Ae^t + 3Ae^t = 16e^t$

$8A = 16 \Rightarrow A = 2$

$\boxed{y = c_1e^{-3t} + c_2e^{-t} + 2e^t}$

b. 9y' +6 y' + y = ¼ t^2 -14

$(3r+1)^2$

$y_h = c_1e^{-1/3t} + c_2te^{-1/3t}$

$y_p = At^2 + Bt + C$

$y_p' = 2At + B$

$y_p'' = 2A$

$18A + 12At + 6B + At^2 + Bt + C = \frac{1}{4}t^2 - 1$

$A = 1/4$ $\quad 12A + B = 0$ $\quad 3 + B = 0 \Rightarrow B = -3$

$18 + 6B + C = 9/2 - 18 + C = -14$

$= 18 - 15 - 9/2 = 4 - 4\frac{1}{2} = -\frac{1}{2}$ $\quad C = -\frac{1}{2}$

$y_p = \frac{1}{4}t^2 - 3t - \frac{1}{2}$

$\boxed{y = c_1e^{-1/3t} + c_2te^{-1/3t} + \frac{1}{4}t^2 - 3t - \frac{1}{2}}$

3. I accidentally left a sheet pan in the oven overnight at 400°. When I removed the pan in the morning it was completely heated to 400°. Ten minutes later it had cooled to 232°. I keep my house at 64°. How "hot" would the pan have been t minutes after I removed it from the oven? How about an hour after it came out of the oven? (Assume Newton's Law of Cooling applies).

$\frac{dT}{dt} = k(T-M) = k(T-64)$

$\frac{1}{T-64}dT = k\,dt$

$\ln(T-64) = kt + C$

$T - 64 = Ce^{kt}$

$T = 64 + Ce^{kt}$

$64 + C = 400$

$C = 336$

$T = 64 + 336e^{kt}$

$232 = 64 + 336e^{10k}$

$168 = 336e^{10k}$

$\frac{1}{2} = e^{10k} \Rightarrow 10k = \ln(2) \Rightarrow k \approx .069$

$T(t) = 64 + 336e^{-\ln(2)t/10}$

$T(60) = 64 + 336e^{-\ln(2)6}$

$= 64 + 336(2^{-6})$

$\approx 69.25°$

69.25°