Section 8.3 Problem 36.

$2x'' + x' + (6-t)x = 0 \qquad x(0) = 3, \ x'(0) = 0$

$2\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2} + \sum_{n=1}^{\infty} a_n n t^{n-1} + \sum_{n=0}^{\infty} 6a_n t^n - \sum_{n=0}^{\infty} a_n t^{n+1} = 0$

$\sum_{k=0}^{\infty} 2(k+2)(k+1)a_{k+2}t^k + \sum_{k=0}^{\infty} a_{k+1}(k+1)t^k + \sum_{k=0}^{\infty} 6a_k t^k - \sum_{k=1}^{\infty} a_{k-1}t^k = 0$

$4a_2 + a_1 + 6a_0 + \sum_{k=1}^{\infty}\{2(k+2)(k+1)a_{k+2} + (k+1)a_{k+1} + 6a_k - a_{k-1}\}t^k = 0$

$4a_2 = -(6a_0 + a_1) \qquad 2(k+2)(k+1)a_{k+2} = a_{k-1} - 6a_k - (k+1)a_{k+1}$

$a_2 = -\frac{3}{2}a_0 - \frac{1}{4}a_1$

$$a_{k+2} = \frac{a_{k-1} - 6a_k - (k+1)a_{k+1}}{2(k+2)(k+1)}$$

$a_0 = 3$

$a_1 = 0$

$a_2 = -\frac{9}{2}$

$a_3 = 1$

$a_4 = 1$

$k=1$

$$a_3 = \frac{a_0 - 6a_1 - 2a_2}{2(3)(2)} = \frac{3+9}{4(3)} = \frac{3(4)}{4(3)} = 1$$

$k=2$

$$a_4 = \frac{a_1 - 6a_2 - 3a_3}{2(4)(3)} = \frac{+27-3}{8(3)} = \frac{24}{24} = 1$$

Alternatively:

$2x'' = -x' + (t-6)x \qquad x(0) = 3\ ; \ x'(0) = 0$

$x'' = -\frac{1}{2}x' + (\frac{1}{2}t - 3)x \qquad x''(0) = 0 + (-3)(3) = -9 \Rightarrow a_2 = \frac{-9}{2!} = -\frac{9}{2}$

$x''' = -\frac{1}{2}x'' + \frac{1}{2}x + (\frac{1}{2}t-3)x' \qquad x'''(0) = +\frac{9}{2} + \frac{3}{2} - 3(0) = \frac{12}{2} = 6$

so $a_3 = 1! = \frac{x'''(0)}{3!}$

$x^{iv} = -\frac{1}{2}x''' + \frac{1}{2}x' + \frac{1}{2}x' + (\frac{1}{2}t-3)x''; \ x^{iv}(0) = -3 + (-3)(-9) = 27 - 3 = 24$

so $a_4 = 1$ also.

It's nice to finally get the same answer both ways!