Differential Equations Test 1 Name KMS

When asked to find a solution to a differential equation, when possible (and only when possible) find an explicit solution. Otherwise, an implicit solution will suffice.

1. To the right are two direction fields. One is for each of the DE's below. Identify which direction field goes with which equation.

a. $\frac{dx}{dt} = \sin(xt)$ Figure 2

b. $\frac{dx}{dt} = x^2 - t$ Figure 1

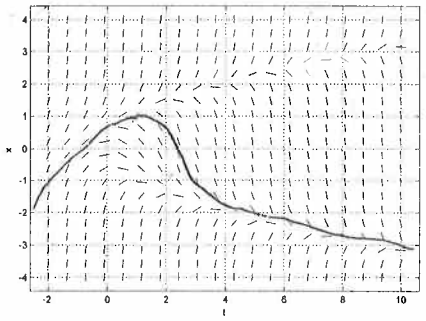

Figure 1

On each direction field sketch a solution to the DE where x(1) = 1.

$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

2. Find a general solution to: $\frac{dy}{dx} = x - y$

$y' + y = x$

$[e^x y]' = xe^x \Rightarrow e^x y = xe^x - e^x + C \Rightarrow \boxed{y = x - 1 + Ce^{-x}}$

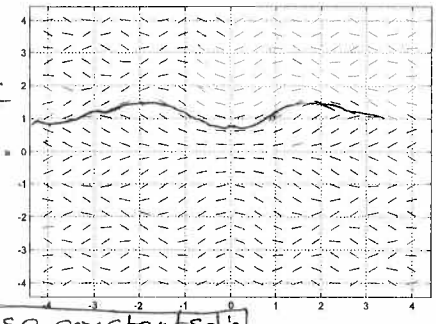

Figure 2

3. Find All solutions to: $\frac{dy}{dx} = (y-2)^2(3x^2 - 1)$ $\boxed{y = 2 \text{ is a constant soln}}$

$\int \frac{1}{(y-2)^2} dy = \int 3x^2 - 1 \, dx$

$-\frac{1}{(y-2)} = x^3 - x + C \Rightarrow y - 2 = -\frac{1}{x^3 - x + C} \Rightarrow \boxed{y = 2 - \frac{1}{x^3 - x + C}}$

4. Find a general solution to: $\frac{dy}{dx} = -\frac{y\cos(xy) + ye^{xy} + 2x}{x\cos(xy) + xe^{xy} + 2y}$ looks exact (?)

$(x\cos(xy) + xe^{xy} + 2y)\,dy + (y\cos(xy) + ye^{xy} + 2x)\,dx = 0$

$\sin(xy) + e^{xy} + y^2 + g(x) \Rightarrow g'(x) = 2x$ so $\boxed{\sin(xy) + e^{xy} + y^2 + x^2 = C}$ Implicit, can't solve for y.

5. Solve the Initial Value Problem: $\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x}$; y(1)=1.

Let $v = y/x \Rightarrow y = xv \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = e^v + v \Rightarrow e^{-v}dv = \frac{1}{x}dx \Rightarrow -e^{-v} = \ln|x| + C$

$e^{-v} = -\ln|x| + C = C - \ln|x|$

$-v = \ln(C - \ln|x|) \Rightarrow v = -\ln(C - \ln|x|)$

$y/x = -\ln(C - \ln|x|)$

$\boxed{y = -x\ln(C - \ln|x|)}$

$1 = -\ln(C) \Rightarrow C = \frac{1}{e}$

$\boxed{y = -x\ln(\tfrac{1}{e} - \ln|x|)}$

6. Lily is another member of a rare, exotic endangered aquatic species who resides at Sea World in Florida. Unfortunately, until recently, like Sammy, Lily was not adequately cared for. Someone allowed the saline concentration in her 100 gallon tank to rise to a dangerous 6%. The new manager (remember her) doesn't want to shock Lily's system by correcting the problem all at once (exotic aquatic species have delicate constitutions) so she plans to continuously add 3% saline solution at a rate of 10 gallons per day while bleeding off tank water at the same rate. Assuming Lily keeps her tank well stirred, how long will it take to bring the saline level to a more reasonable 4.5%? 6 days 21 hours 36 min. ≈ 6.9 days Mathematically speaking, how long would it take until the level was 3%? Never, it approaches 3 as $t \to \infty$.

$S(0) = 6 \qquad \frac{dS}{dt} = (10)(.03) - \frac{S}{100}(10) = .3 - \frac{S}{10} = \frac{3-S}{10}$

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so $|S-3| = 3-S$

$\int \frac{10}{3-S} dS = \int dt \Rightarrow -10\ln|3-S| = t + C \Rightarrow \ln|3-S| = -\frac{t}{10} + C \Rightarrow |3-S| = Ce^{-t/10}$

$\Rightarrow S - 3 = Ce^{-t/10} \Rightarrow S = 3 + Ce^{-t/10} \quad C = 3. \text{ So } S = 3 + 3e^{-t/10} = 4.5 \Rightarrow 1.5 = 3e^{-t/10}$

$\Rightarrow .5 = e^{-t/10} \Rightarrow \ln(.5) = -t/10 \Rightarrow t = 10\ln(2) = 6.9$

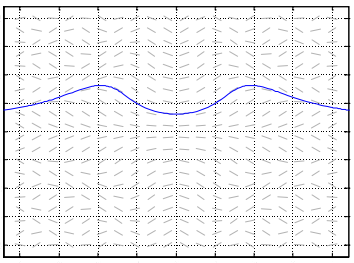

Figure 2

From Problem 1:
Here are approximate solutions drawn by Matlab. They seem pretty close to what I drew by hand...I couldn't get the cursor right on (1,1) so they are within .05 (I'm pretty sure) of 1 for each coordinate.

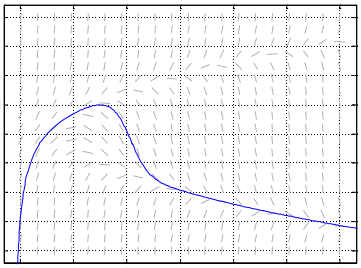

Figure 1