Test 2

Part A

1. $2r^2+3r+1=0$ $\boxed{y=c_1e^{-x}+c_2e^{-x/2}}$

$(2r+1)(r+1)=0$

$r=1/2 \quad r=-1$

2. $r^2-10r+25=0$ $\boxed{y=(c_1+c_2t)\,e^{+5t}}$

$(r\mp 5)^2=0$

double root $r=-5$

3. $4r^2+1=0$ $\boxed{y=c_1\sin(\tfrac{1}{2}x)+c_2\cos(\tfrac{1}{2}x)}$

$r=\pm\frac{1}{2}i$

4. $y_p=Ae^x=y_p'=y_p''$ $\boxed{y_p=e^x}$

$2Ae^x+3Ae^x+Ae^x=6e^x$

$2A+3A+A=6A=6$

5. Variation of parameters

$V_1=-\int\frac{g(x)\,y_2}{aW}\,dx \quad V_2=\int\frac{g(x)\,y_1}{aW}$

$y_1=e^{-x} \quad y_1'=-e^{-x}$

$y_2=e^{-x/2} \quad y_2'=-1/2\,e^{-x/2}$

$W=y_1y_2'-y_1'y_2=-\frac{1}{2}e^{-x}e^{-x/2}+e^{-x}e^{-x/2}$

$=\frac{1}{2}e^{-3x/2}$

$V_1=-\int\frac{1/(1+e^x)\,e^{-1/2x}}{2(1/2e^{-3/2x}}\,dx$

$=-\int\frac{e^x}{1+e^x}\,dx=-\ln(1+e^x)$

$V_2=\int\frac{1/(1+e^x)\,e^{-x}}{e^{-3/2x}}\,dx=\int\frac{e^{1/2x}}{1+e^x}\,dx$

$=2\int\frac{\frac{1}{2}e^{1/2x}}{1+(e^{1/2x})^2}\,dx=2\arctan e^{x/2}$

$y_p=V_1y_1+V_2y_2$

$=-e^{-x}\ln(1+e^x)+2e^{-1/2x}\arctan e^{x/2}$

6. $\boxed{y_p=2e^x-4e^{-x}\ln(1+e^x)+8e^{-1/2x}\arctan(e^{x/2})}$

7. $y_p=Ae^{-5x}$

$y_p'=-5Ae^{-5x}$

$y_p''=+25Ae^{-5x}$

$y_p''-10y_p'+25y_p=(25+50+25)Ae^{-5x}=e^{-5x}$

so $A=\frac{1}{100}$

$y_p=\frac{1}{100}e^{-5x}$

Part B

1) $4x''+x=0 \Rightarrow x=c_1\sin 1/2t+c_2\cos 1/2t$

$x'=\frac{1}{2}c_1\cos 1/2t-1/2\,c_2\sin 1/2t$

$x(0)=c_2=3 \quad x'(0)=1/2\,c_1=2 \Rightarrow c_1=4$

$A=\sqrt{c_1^2+c_2^2}=\sqrt{9+16}=\sqrt{25}=5$

the Amplitube is 5.

2) $b^2-4mk<0 \Rightarrow b^2-16<0 \Rightarrow b<4$

damping coeficients less than 4 result in underdamped motion.

Part C

(1) Let $v=x'$ then

$v'=\sin(t)-xv-\cos t\,x$

$x'=v.$

2. $(D+3)[x]-2y=0$

$+5x \quad +(2D-3)[y]=0$

$(2D-3)(D+3)[x]+10x=0$

$2D^2+3D-9+10\,[x]=0$

$(2D^2+3D+1)[x]=0 \Rightarrow x=c_1e^{-t}+c_2e^{-t/2}$

$2y=(D+3)[x]$

$2y=-c_1e^{-t}-\frac{c_2}{2}e^{-t/2}+3c_1e^{-t}+3c_2e^{-t/2}$

$=2c_1e^{-t}+\frac{5}{2}c_2e^{-t/2}$

$\boxed{y=c_1e^{-t}+\frac{5}{4}c_2e^{-t/2}}$