Teaching Inequalities: A Hypothetical Classroom Case

Pre-case exercises:

1. Solve the inequality $3x + 1 \leq 7$ without performing any operations that re-arrange the terms in the existing inequality. Instead, use a number line as a tool to solve the problem.
2. Describe how $3x + 1 \leq 7$ is similar to 3x + 1 = 7. Describe how they are different.
3. How is the solution to $-3x + 1 \leq 7$ different from the solution to $3x + 1 \leq 7$? Why?
4. Write a word problem for the inequality $3x + 1 \leq 7$.

Scenario:

Mr. Smith is looking for ways to get his class to understand inequalities. He has been unhappy with his past practices, but he has not been sure what he should change in order to help deepen students' understanding. He has approached you, as a colleague who teaches mathematics, to observe one of his lessons and give him some feedback on his teaching. You have agreed to do an observation for him. The next section describes some of the events that occurred during the lesson you observed. As you read through the following set of events, write down some thoughts that you might share with Mr. Smith when you sit down with him to discuss the lesson after school that day.

The lesson:

As Mr. Smith's students entered the room, they found a set of warm-up exercises on the board. Students were to solve the following five equations in the warm-up: (i) 3(x + 1) = 21, (ii) 3(a - 3) = 2(a + 4), (iii) 3(y - 3) = 6, (iv) 3(a - 1) = 4(a - 15), (v) 2(2w + 5) + 2w = 46. After a few minutes of class work time, Mr. Smith asked five students to come up to the board to solve each of the equations. When the students had finished writing their work on the board, Mr. Smith asked if there were any questions. When no one asked any questions, he decided it would be OK to move on to the next portion of the lesson.

Mr. Smith began the next portion of the lesson by asking students, "What do you think of when I say the word 'inequality'?" Several students shot their hands up to volunteer responses. Some of the student responses included:

- "It is a Pac-man between two numbers"
- "It is like an equation but it has a pointy symbol instead of an equal sign"
- "You do an inequality almost like how you do an equation"
- "You can solve an equation but not an inequality"

Unsatisfied with the set of student responses he received, Mr. Smith continued the lesson. He was especially annoyed with the fact that students referred to the inequality sign as a "Pac-man" rather than using its proper name. He decided to give them some simple examples of uses inequality signs. He said, "In the United States, you have to be at least 18 years old to vote. We could represent that with an inequality symbol by writing, $you \geq 18$." Mr. Smith went on to give another example, "This semester, you will get at most 250 reward tickets from me for being good in class. 250 is the maximum any one

person can get. We could represent that with an inequality symbol by writing, $you \leq 250$." He then went on to point out that when a problem contains the phrase "at least," you should use the "≥" sign, and when a problem contains the phrase "at most," you should use the " ≤" sign.

Next, Mr. Smith asked students to work in groups of four to solve some problems. He said, "Imagine that there are 15 students who all want to earn money by putting in hours working at the school store this week. The school can pay for at most 60 total hours this week. How many hours should each person work if the hours are split fairly?" He then put the inequality $15h \leq 60$ up on the overhead projector and then asked students to solve and graph it. Students had solved inequalities the previous year, so some of them knew that graphing inequalities had something to do with open dots, closed dots, and arrows on a number line. Mr. Smith circulated about the room as the groups worked in order to remind them of when to use open dots and closed dots. As he circulated, he told them that the arrow on the graph should point in the same direction as the arrow in the inequality sentence (so the arrow on the graph of $15h \leq 60$ would point to the left).

When most of the groups had finished working on the inequality on the overhead projector, Mr. Smith called the class back together so they could watch one of the students, Keisha, solve it for everyone. Keisha drew a number line on the transparency, put a closed dot on the four, and then drew an arrow coming from the dot and going to the left. When Mr. Smith asked the student to explain her thinking, she said, "First, I pretended that there was an '=' sign between the 15h and the 60 so I could treat it like an equation. Then I divided both sides by 15. I got $h \leq 4$. That means each student can work at most 4 hours each, because ′ ≤ ′ means 'at most.' Then I put a solid dot on the four because there is a line underneath the 'at most' symbol. Then I drew an arrow to the left because the 'at most' symbol is pointing to the left." Pleased that Keisha had talked so much about how she solved the problem, Mr. Smith thought it would be good to give the students a more challenging problem.

Mr. Smith put the inequality $\frac{x}{-3} \leq 4$ on the overhead next. He asked students to work with their groups again in order to solve and graph it. They quickly came up with the solution $x \leq -12$. Mr. Smith called a student, Jasmine, up to the overhead projector to graph the solution. She drew a number line, put a solid dot on the 12, and then drew an arrow to the left. After Jasmine sat back down, Mr. Smith went up to the graph she had drawn pointed to "-15" on the number line. He asked students to put "-15" in for *x* in the original inequality. When students did so, they got $5 \leq 4$. When Mr. Smith asked students if it was true that 5 is less than or equal to 4, a few said "no." He asked them to talk in their groups for a few minutes about what had happened in this situation. After students spent a few minutes squirming in their seats and talking about who had been voted off of *American Idol* the previous night, Mr. Smith asked the class to quiet down, and to direct their attention back toward him. He wrote on the overhead projector, "When you divide or multiply by a negative in an inequality, you need to flip the sign." He told students to write the rule in their notebooks, because they would be using it for homework. As he was writing on the projector, the bell signaling the end of the class period rang. He quickly wrote the homework assignment on the projector, and told students to copy it down before leaving class.

Questions for reflection and discussion:

1. What were the strongest points of Mr. Smith's lesson? What were the weakest points?
2. Do you think that Mr. Smith's warm-up activity was a good lead-in to the concepts he was trying to teach in the main portion of the lesson? Why or why not?
3. Do Mr. Smith's students accurately reflect the way that most students think about inequalities? Why or why not?
4. Does Mr. Smith's rule about translating the phrases "at least" and "at most" into inequality symbols work all of the time? Why or why not?
5. Was Mr. Smith's example of 15 students working in the school store a good word problem to use to illustrate $15h \leq 60$? Why or why not?
6. Critique Mr. Smith's approach to teaching (or reminding) students how to graph inequalities.
7. Critique Keisha's work on the problem she demonstrated at the front of the room.
8. Critique Mr. Smith's use of group work. Was his use of group work appropriate for this lesson? Why or why not?
9. What steps could Mr. Smith have taken in order to lead students toward discovering the rule for "flipping" an inequality sign when multiplying or dividing by a negative?
10. Mr. Smith made extensive use of the overhead projector during this lesson. What are the advantages and disadvantages of using an overhead projector as a teaching tool?
11. Do you think that Mr. Smith's students will retain the material they learned during this lesson for a long period of time? Why or why not?
12. What should students know about inequalities at various different grade levels (grades 5-8)? How can teachers build on what students learn about inequalities at each grade level?