Developing Fraction Proficiency in Fourth Grade Mathematics

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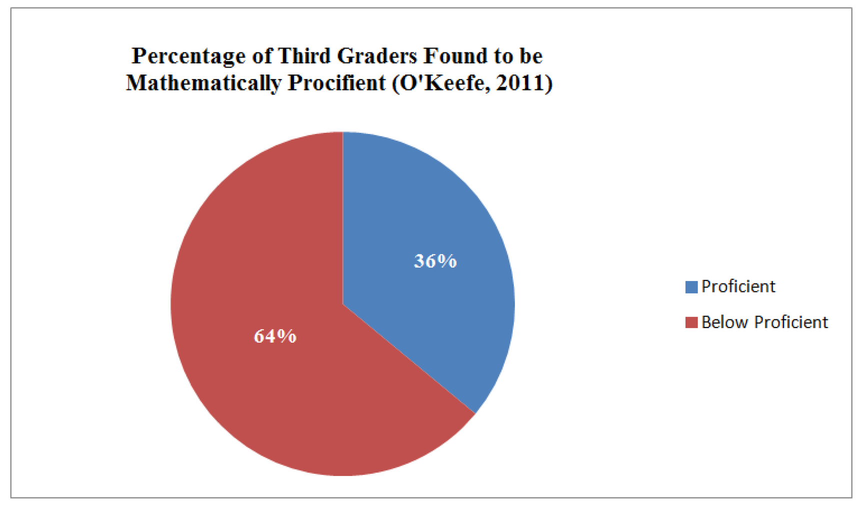
Introduction:

Research confirms that third grade students in the United States have difficulty with multiple mathematical topics they are expected to know upon entering the fourth grade. For instance, O'Keefe (2011) found that only 36% of third graders in the District of Columbia were mathematically proficient. This situation has yet to show any compelling signs of improvement (O'Keefe, 2011). Nationwide, only 41% of fourth grade students performed at or above the "proficient" level in mathematics on the most recent National Assessment of Educational Progress (National Center for Educational Statistics, 2013). These statistics are troubling, considering that mathematics proficiency by the end of third grade is vitally important for success in later grade levels (National Council of Teachers of Mathematics, 2000).



Fractions pose one of the biggest challenges to third graders in developing mathematical proficiency. The Common Core State Standards state that by the end of third grade students should have a strong foundation of fraction representation, inequalities and equivalency, and fraction proximity of a whole and half a unit (National Governors Association for Best Practices & Council of Chief State School Officers, 2010).

The purpose of this study was to explore and develop four rising fourth grade students' thinking about fractions. The specific research questions were:

1. What difficulties do the students have with fractions concepts proscribed by the Common Core?
2. What types of representations and teaching strategies are effective in helping the students develop mathematical proficiency with fractions?

References:

National Center for Education Statistics. (2013). *A first look: 2013 mathematics and reading.* Retrieved from http://nces.ed.gov/nationsreportcard/subject/publications/main2013/pdf/2014451.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Governors Association for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Retrieved from http://www.corestandards.org/.

O'Keefe, Bonnie. "Third Grade Proficiency in DC: Little Progress (2007-2011)." *DC Action for Children.* N.p., n.d. Web. 4 Aug. 2014. http://files.eric.ed.gov/fulltext/ED538791.pdf.

Theoretical Framework:

Conceptualizing Mathematical Proficiency

Mathematical proficiency can be conceptualized as five interwoven strands. *Adding it Up* (Kilpatrick, Swafford, & Findell, 2001, p. 116) described the five strands in detail:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

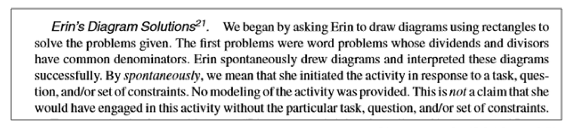
Hypothetical Learning Progression for Fractions

In defining the idea of learning progression, Battista (2011) states, "General learning progression describes students' various ways of reasoning about a topic, irrespective of curriculum; it focuses on understanding and reacting to students' current cognitive structures." (p. 7)
The Common Core State Standards Writing Team (2013) constructed a hypothetical progression for learning fractions that is based upon past research. Key points from this learning progression that helped guide our instructional interventions were:

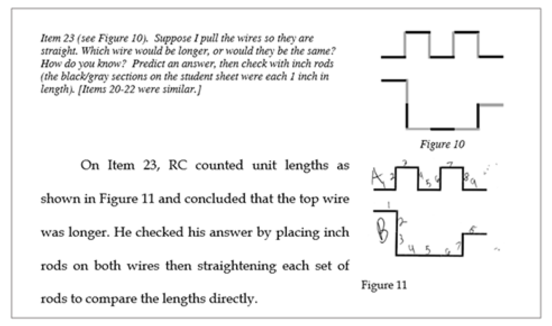
- Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole Into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is 1/4 of the whole, and 4 copies of that part make the whole (p. 3).
- Grade 3 students do some preliminary reasoning about equivalent fractions, in preparation for work in Grade 4. As students experiment on number line diagrams they discover that many fractions label the same point on the number line, and are therefore equal; that is, they are equivalent fractions (p. 4).
- Previously, in Grade 2, students compared lengths using a standard measurement unit.2.MD.3 In Grade 3 they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions (p. 4).
- Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater (p. 5).
- As students move towards thinking of fractions as points on the number line, they develop an understanding of order in terms of position. Given two fractions—thus two points on the number line— the one to the left is said to be smaller and the one to right is said to be larger (p. 5).

Instructional Approach

Simon et al. (2010) suggested that "individuals have the capacity to learn through their mathematical activity, (and) the possibility exists to engineer a sequence of tasks that promotes the learning of individuals through their engagement with such a task sequence." (p. 72). Some tasks that past studies have shown to be valuable to task sequences involving fraction learning are:



Simon et al. (2010, p. 25)



Battista (2011, p. 28)

In the tasks displayed above, the students are able to use visuals to arrive at their final answers. With our students especially, visual aids proved to be very useful when solving problems. When able to use their own hands to create manipulatives or draw them out, it gives the child a better sense of what the term fraction really means. Incidentally, the students are able to get a better grasp of the concepts and the probability of retention is much higher.

References:

Battista, M. T. (2011). Conceptualizations and issues related to learning progressions, learning trajectories, and levels of sophistication. *Montana Mathematics Enthusiast, 8*(3), 507-570.

Common Core State Standards Writing Team. (2013). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 3-5, Number and Operations – Fractions.* Tucson, AZ, Institute for Mathematics and Education, University of Arizona.

Kilpatrick, J., Swafford, J., Findell, B., & National Research Council (eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Simon, M., Saldanha, L., McClintock, E., Akar, G.K., Watanabe, T., & Zembat, I.O. (2010). A developing approach to studying students' learning through their mathematical activity. *Cognition and Instruction, 28*(1), 70-112.

Methodology

Participants and procedure

The four students in our study recently finished third grade. They each came in to our classroom once a week for seven one hour session plus the introductory and concluding interviews. We had two females, Jordan and Daisy as well as two males, Jay and Nick (pseudonyms). All four students participated in all of the scheduled lessons and interviews.
The instructional goals for our lessons were chosen from the Common Core State Standards (CCSS) (National Governor's Association for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010):

CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

CCSS.MATH.CONTENT.3.NF.A.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

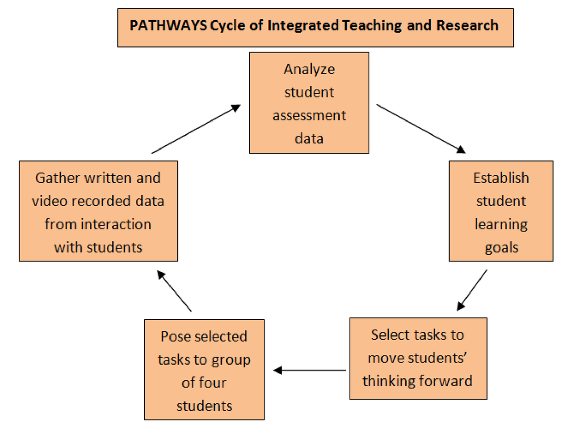
CCSS.MATH.CONTENT.3.NF.A.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

CCSS.MATH.CONTENT.3.NF.A.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

References:

National Governor's Association for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Author. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Below is the cycle in which we went about our project. We started by analyzing the students' assessment data to determine the weaknesses and strengths of each student. We then used what we learned to establish the learning goals we wanted to accomplish. In order to achieve these goals, we selected the tasks that would help get the students thinking in an inquiry based way. After putting these tasks into play, we were able to use the recorded data that was gathered to move forward in our teaching and research.



Data Gathering and Analysis

On the very first week of the project, the students were given both a written and a verbal/interview style pre-assessment so that we could gauge their level of comprehension. Eight weeks later, students were also given a post-assessment that consisted of the same set of interview and written questions. Comparing the results of the pre-and post-assessments helped us determine the effectiveness of our teaching. During the interview process in both the pre and post assessments, we were able to sit down one-on-one with each child and listen to them explain, in their own words, the ways in which they were able to solve the given problems. While we were unable to give them advice or inform the students whether their answers were correct or incorrect, we were able to constructively encourage them to think about what methods they could employ. Through this process we were given the opportunity to view certain questions from the child's perspective and actually begin to understand the various approaches to problem solving. For example, in one post-assessment interview question, the children were given a problem that instructed them to label the pictures (of candy bars) from least to greatest in terms of amount eaten. In another interview, students were asked to plot a point on a number line that was equivalent to 3/4 and explain their reasoning. They were given a blank number line so that they could choose from several different points and in turn, think outside of the box to solve the problem. One of the other questions addressed on the interview assessment pertained to equivalency in fractions. On this task, the students could either draw a picture or a number line to reach their conclusion. These open-ended sort of questions challenged the students to use different methods to show proof of their responses.

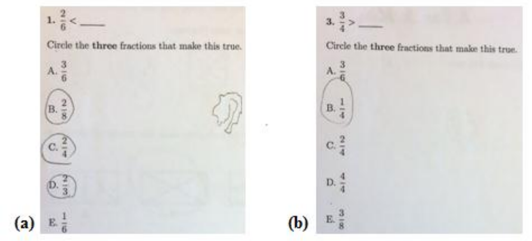
Data Gathering and Analysis Procedures Followed for Each Interview and Instructional Session

- Video record 1-hour session
- Retain all student work samples; archive work samples and videos electronically
- Watch video and transcribe verbatim.
- Review transcript and code segments using the 5 Strands of Mathematical Proficiency model
- Summarize students' attainment of each strand of mathematical proficiency and make data-based conjectures about how to enhance students' thinking to guide design of the next lesson

Empirical Teaching and Learning Trajectory:

Initial Assessment Results

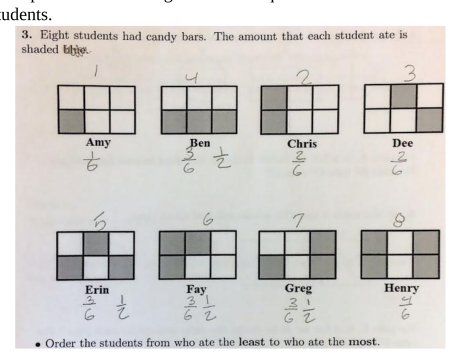
The results from the completed initial assessment were overall very inconsistent. While some students struggled in one area, others had a difficult time in another. The most striking of all results came from Daisy. While going through her answers, it was rare that she even completed the problems. Instead, she wrote, "I do not know" as her final answer. This suggested that she needed to develop along all five strands of mathematical proficiency in regards to fractions. Other students were very inconsistent with their answers on the inequality problems. For problems asking them to circle three correct responses (see below), the students would each get each partially correct or they would get them all and then miss the next inequality question that was only slightly different from the first. For example, the student below was partially correct in his response to the first problem (a). However, when the problem was changed slightly, he could not come up with the correct answers and only selected one (b).



(a) (b)

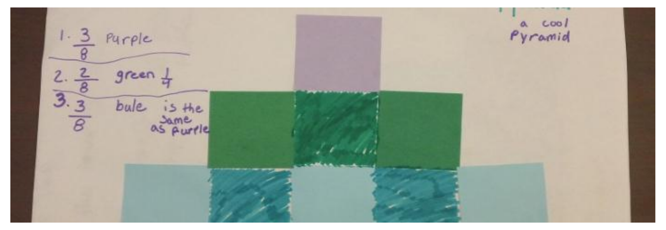
This example shows the weakness in strategic competence that this student initially had. Lastly, when given another list of fractions, the students each struggled determining which was closest to one half and which was closest to one whole. This immediately informed us that the students needed a better conceptual understanding.

Aside from weaknesses, it was also apparent that the students had some combined strengths. The students were asked to order and label a set of fraction pictures from least to greatest. Each student completed this with ease. They all determined the amount of shaded pieces as well as noting that the picture displaying 3/6 could be rewritten as 1/2. This problem indicated that the students had the ability to be procedurally fluent in naming fractions. It also showed some beginning conceptual understanding of fraction equivalence. Below is an example of this problem completed by one of the students.



Instructional Sessions (Weeks 2-3)

Our first lesson, which was week 2 of the program, was entitled "fraction-art." Using pre-cut fraction tiles, we had the students create their own shape/design that included a few fraction-guidelines we had previously implemented. For example, students produced the following designs in response to the task:



In the next lesson, we had the students construct their own fraction walls so that they could develop conceptual understanding of what the different fractions were and what fractions they were equivalent to. A sample student fraction wall is shown below:

Upon completion, we had the students label each part of their wall, and then moved onto a set of questions for them to answer where they were able to reference their newly finished project.

After putting together their fraction walls, students demonstrated *strategic competence* when we asked them to give us a fraction equivalent to ¼. They were able to not only tell us the answer, but also show us how they arrived at the solution. They also demonstrated *adaptive reasoning* in ultimately using the fraction walls as tools for proof and justification. We believe that the activities from the first two weeks also helped students develop a *productive disposition* toward mathematics. The children could now see fractions as more than just "math." Rather than dreading these numbers, they now could point out realizations from their every-day encounters and relate it back to what they were learning.

Instructional Sessions (Weeks 4-6)

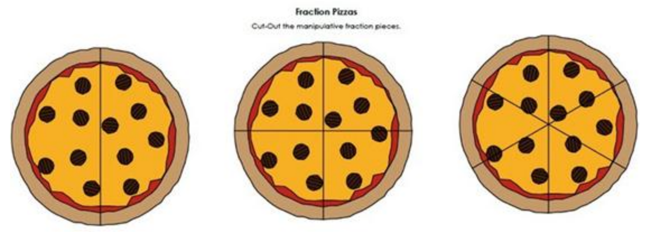
During weeks 4-6, we incorporated dice activities to add the element of random selection. The intent of this activity was to get the children thinking more about equivalency and having them identify fractions that fit in a more rapid, yet accurate manner, thus exhibiting procedural fluency while completing this task. Pictured below is the table the children were given:

Fraction Dice!

Equivalencies

Fraction Rolled	Equivalent Fraction

Next, we encouraged students to think about circular area models for fractions. We designed and executed a "pizza" themed lesson. Part of the template used is shown below:



In order to work with fractions in the pizza context, the children exercised skills in *strategic competence*. Rather than working with the usual, fraction-bar style manipulatives, the student's now had to convert their thinking to represent these proportions in a more complex way.

In another lesson, we focused on individual work with the students rather than our typical group-work procedures. This allowed us to see where each child stood in terms of *conceptual understanding*. For instance, Jay was given the fraction 4/8 and asked to find one or more fractions equivalent to it, and draw a picture to justify. After completing the individual work, the students were given the opportunity to share their work with the rest of the children in the group For example, Daisy solved the sample problem by drawing individual "cupcakes" and then splitting them to arrive at her answer and then shared the strategy with the rest of the group. The purpose of this lesson was to provide the children with more than one way to solve problems, and enhance their *productive disposition* in regards to their work.

Instructional Sessions (Weeks 7-8)

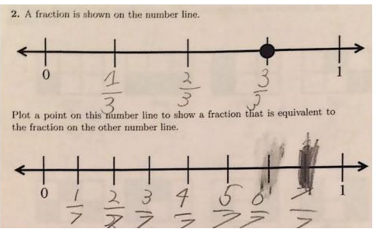
The final two weeks of instructed focused on a more in-depth exploration of fractions and all of their components. We reintroduced the number line to the students in one of our lessons. We did an activity that involved placing fraction clothespins in order from smallest to largest on a coat hanger. This was a way for the students to employ *adaptive reasoning* tendencies as they labeled their coat hanger number line. The students were able to order their coat hanger quickly with fractions containing the same denominator, while fractions with varying denominators took a tad longer. Below is an example of the activity in process:

Source: <http://love2learn2day.blogspot.com/>

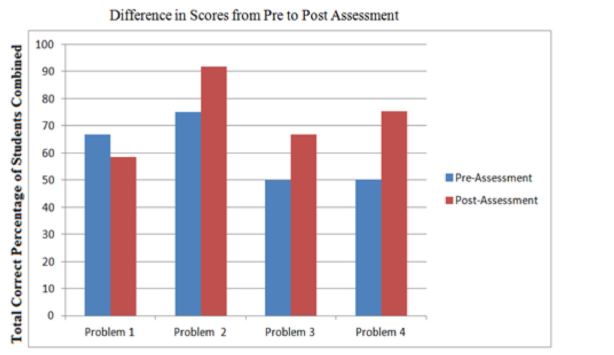
In another lesson entitled "Fraction Avenue," we witnessed the students displaying a *productive disposition* toward their work. We gave the students sample scenarios that consisted of "paving" different driveways with pre-cut fraction-strips. Since the pupils could relate this situation to practices they have actually seen outside of school, it made it easier for them to associate the activity with truly being realistic. In addition to being able to see the relevance in this form of mathematics, the children demonstrated *strategic competence* by solving these problems and creating the correct fractions to represent the amount of driveway paved. It was challenging, however, for them to answer the question of which driveway was closest to being done. The picture below shows the materials used to conduct this lesson:

Post Assessment Results

One weakness that was consistent for all four students in the post assessment was the ability to interpret number line representations of fractions. The students were asked to determine a fraction on a number line as well as plot a point on another number line in which represented the same fraction. A sample student submission (below) shows a prevalent difficulty in solving this problem successfully. In the example, the student believed the hash marks on the first number line to be a length of one-third each because there were three marks between zero and one.



Students generally did better, however, on post-assessment problems requiring the comparison of fractions in inequality statements. Below is a bar graph showing the percentage of correct responses given for each of the four inequality items.



Improvement was also shown in the problems asking to determine which, out of a list of fractions, was closest to one half and one whole. For each of these problems, there was an additional student that got the answer correct. The improvement in these problems suggested stronger conceptual understanding of fractions, as they required students to reason about the relative sizes of the quantities given. Also notable is that Daisy showed evidence of improved productive disposition by completing several problems on the post-assessment successfully, whereas on the pre-assessment she did not attempt any items.

Reflection and Discussion

CCSS.MATH.CONTENT.3.NF.A.2 states: [Students should] Understand a fraction as a number on the number line; represent fractions on a number line diagram. In our experience over the course of this project, students had the most trouble with this particular standard. Students encountered problems when asked to label number lines that were completely blank as well as those that were partitioned.

We observed that it is important to emphasize several key ideas when teaching fractions. Stressing the importance of equal parts and the idea of having the "same whole" is essential when comparing fractions. If students begin to stray away from these notions, it is important to remind them of their significance to get them back on track. Keeping the students interested in their work is equally critical. We were generally able to design engaging contexts to capture students' interest. Visual models that allowed the students to reason about relative sizes of fractions and fraction equivalence were also helpful at various points.

A word of advice to future educators and those that are transitioning into teaching the learning progression: While it is important to make sure you are encompassing all aspects of the fraction curriculum, be sure to take your time. Fractions are not to be learned in one day. It may seem frustrating to go over the same concepts for days, or even weeks, but it is imperative that students get these concepts committed to memory. For example, be sure to repeatedly emphasize the idea of the "whole." When comparing fractions it is mandatory for the students to realize that the whole is the same size. Going along with this notion, the students must also make sure to reference equal parts when comparing their pieces to the whole. This is necessary to ensure accuracy when arriving at a solution. Additionally, plan lessons that are relevant to the kids' lives. When you can relate mathematics to reality, the students' are much more likely to comprehend the information.