Products of 3rd Grade Multiplicative Thinking and Reasoning

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Introduction

- Changes prescribed by the Common Core State Standards
  - From memorization to deeper Conceptual Understanding
  - Students demonstrate the process of completing the problem
  - Use of words or diagrams

- Multiplication in the Common Core
  - Mastery begins in 3rd grade
  - Crucial skill
  - Time consuming

Introduction

- Our goal for the research
  - Gain understanding of students' thinking about multiplication
  - Develop students' understanding

- Guiding Research Question:

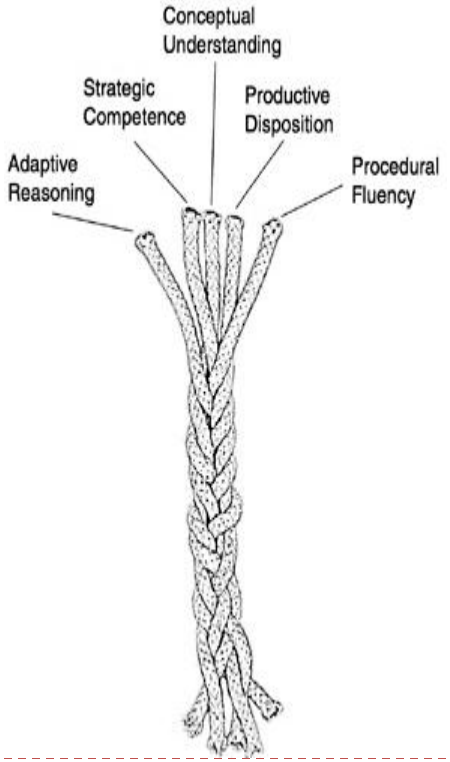
How can students' mathematical proficiency be developed in regard to multiplicative thinking and reasoning?

Theoretical Framework

- Learning Progressions
  - Outlined by Common Core State Standards Writing Team (2011)
  - 2 main focuses for multiplication in Grade 3
    - Equal sized groups
    - Array Representations
  - Student representations and solutions categorized into three levels
    - Level 1- representing the entire amount
    - Level 2- skip counting to solve tasks
    - Level 3- using higher multiplicative properties

Theoretical Framework

- Five Strands of Mathematical Proficiency (Kilpatrick, Swafford, & Findell, 2001)
  - What is needed for learners to fully develop mathematical thinking
  - Interdependent and intertwined strands
    - Conceptual Understanding
    - Procedural Fluency
    - Strategic Competence
    - Adaptive Reasoning
    - Productive Disposition



Theoretical Framework

Review of educational articles

- *Teaching for Mastery in Multiplication* (Wallace & Guganus, 2005)
  - Using meaningful ideas and scenarios
  - Build connections between concepts
  - Use manipulatives and other representations to solve problems
- *Direct Modeling and Invented Procedures. Building on Students' Informal Strategies* (Chambers, 1996)
  - Direct model
    - Using physical objects
  - Invented algorithms
    - Reveal students' sense making

Methodology- Participants and Procedure

Student Population:

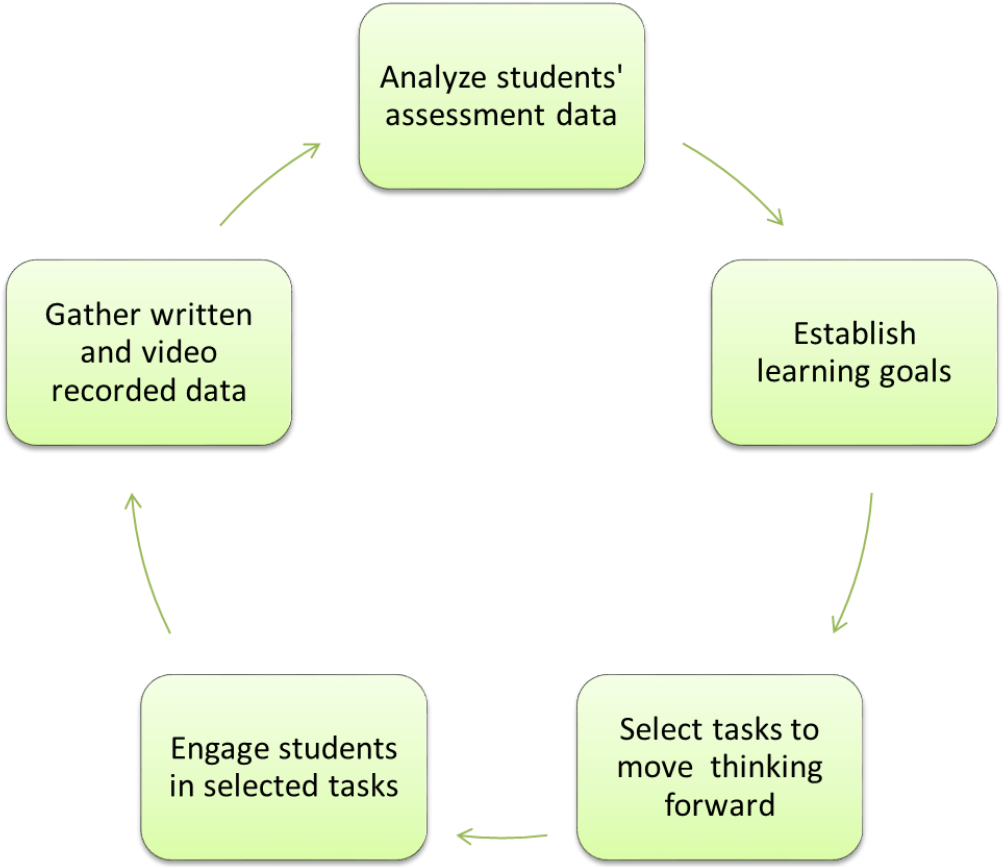
- Students finishing 3rd grade
- 4 students
- Pseudonyms of participants-
  - Tess, Gabbie, Jake, Earl
- Participation rate
- Pre and Post assessment
- Seven 1-hour instructional sessions

Methodology- Participants and Procedure

Common Core State Standards for Mathematics

- CCSS.MATH.CONTENT.3.OA.A.1 - Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.
- CCSS.MATH.CONTENT.3.OA.A.3 - Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- CCSS.MATH.CONTENT.3.OA.A.4 - Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- CCSS.MATH.CONTENT.3.OA.B.5 - Apply properties of operations as strategies to multiply and divide.

Methodology- Participants and Procedure



- PATHWAYS Cycle of Integrated Teaching and Research

Methodology- Data Gathering and Analysis

Pre and Post Interview Protocol

- Written assessment
  - 30 minutes- completed individually
- Clinical interview
  - 30 minutes- completed with undergraduate
  - Examine student thinking through answers and discussion

Methodology- Data Gathering and Analysis

- A few examples of questions are listed below

Ten rows of snails. Four snails in each row. How many snails?

There are four boxes of crayons. Each box has 10 crayons in it. How many total crayons are there?

8 equal rows of cans, 48 total cans. How many cans in each row?

There are 3 tables in Mrs. Potter's art classroom. There are 2 students sitting at each table. Each student has a box of 5 colored pencils.

(A) How many colored pencils are at each table?

(B) How many colored pencils do Mrs. Potter's students have in total?

Methodology- Data Gathering and Analysis

Procedures used in the Research:

- Video Recording
- Transcribing
- Analyzing the interview
- Lessons
- Student work samples

Empirical Teaching and Learning Trajectory:

Next we will discuss:

- Initial Assessment Results
- Instructional Cluster 1
- Instructional Cluster 2
- Instructional Cluster 3
- Post Assessment Results

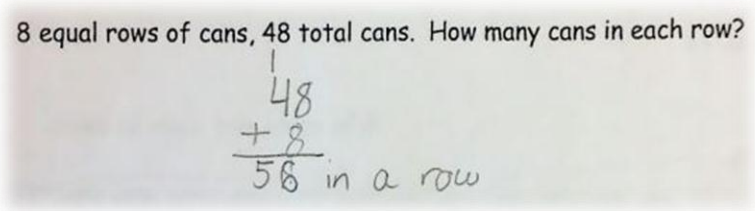
Initial Assessment Results

Based on the clinical interview and written assessment and connected to the Five Strands of Mathematical Proficiency

- Wide Range of Mathematical Proficiency
- Working towards Third-Grade Standards

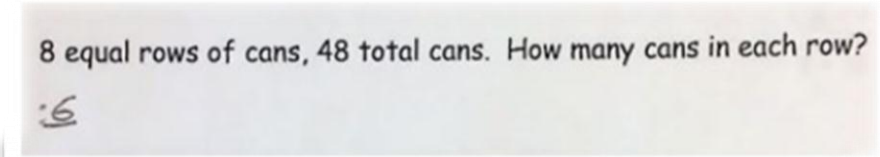
Initial Assessment Results

- Earl and Gabbie- weakness in *Conceptual understanding* of multiplication

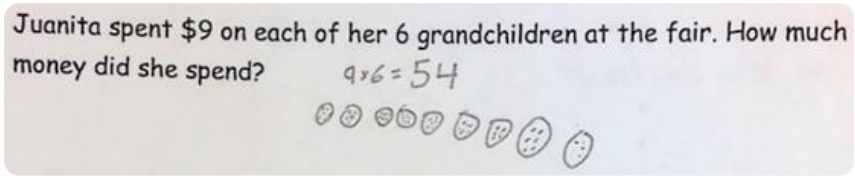


- Gabbie- limited *Productive Disposition* based on confidence approaching problems

- Jake- Strength in *Conceptual Understanding* and *Procedural Fluency* relating to multiplication



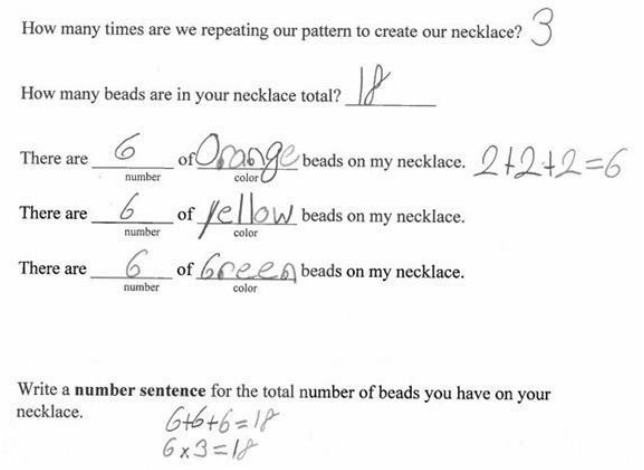
- Some students- strength in *Strategic Competence* through representations



Instructional Cluster 1

Focused on equal sized groups and repeated addition

- Lesson 1
  - Students created a bracelet using a pattern. Explored the number of total beads, as well as each color.



How many times are we repeating our pattern to create our necklace? 3

How many beads are in your necklace total? 18

There are 6 (number) of Orange (color) beads on my necklace. 2+2+2=6

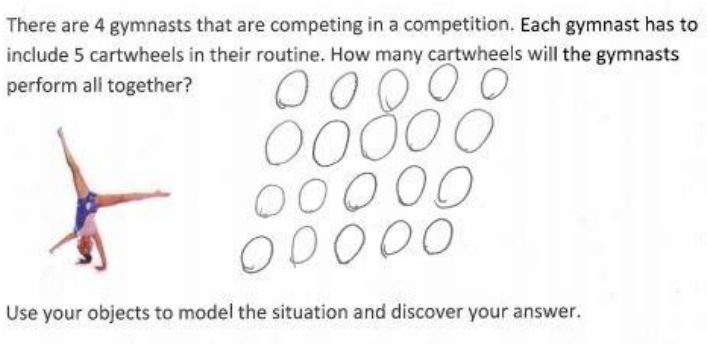
There are 6 (number) of yellow (color) beads on my necklace.

There are 6 (number) of green (color) beads on my necklace.

Write a **number sentence** for the total number of beads you have on your necklace.

6+6+6=18
6x3=18

- Lesson 2
  - Word problems involving equal sized groups of object. Explored the number of total objects.



There are 4 gymnasts that are competing in a competition. Each gymnast has to include 5 cartwheels in their routine. How many cartwheels will the gymnasts perform all together?

Use your objects to model the situation and discover your answer.

Instructional Cluster 1

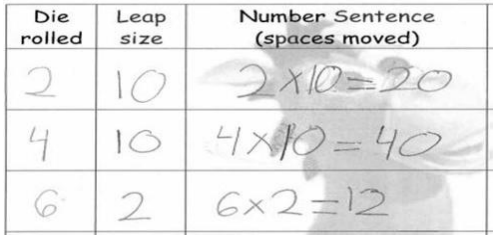
- Lesson 1 (noteworthy observations below)
  - Gabbie- working on concept of equal size groups
  - Tessa- identifying total number and explaining it
  - Jake- recall of multiplication
  - Earl- interesting representations of total number

- Lesson 2 (noteworthy observations below)
  - Jake- comfortable solving problems
  - All students- efficiency in skip counting recognized
  - Tessa- using rectangular array

Instructional Cluster 2

Focused on skip counting, using game board idea to emphasize the connection to multiplication.

- Lesson 3
  - Introductory word problem
  - Board game on floor, skip counting by 2's and 5's
    - Observing student progress through game
- Lesson 4
  - Board game on table, skip counting by 2, 3, 4, 5, 6, and 10
  - Number sentences for place on board and spaces moved



Die rolled	Leap size	Number Sentence (spaces moved)
2	10	2x10=20
4	10	4x10=40
6	2	6x2=12

Instructional Cluster 3

Focused on array representations

- Lesson 5
  - 100 Hungry Ants book
  - Arranging 100 into different arrays
- Lesson 6
  - Array representations of 24
  - Cutting out different arrays and corresponding number sentences
  - Discussion of commutative property
- Lesson 7
  - Problems in division format
  - Review of strategies used throughout experience

Instructional Cluster 3

- Lesson 5 (noteworthy observations below)
  - Pattern seeking
- Lesson 6(noteworthy observations below)
  - Earl could explain his representations and equation
  - Jake showed flexibility with Commutative Property of Multiplication
- Lesson 7(noteworthy observations below)
  - Gabbie was able to solve new problems
  - All students could explain representations

There was a big family of 24 mice that were looking for a new home to live in. They found 4 separate holes to live in.

If the mice wanted an equal number of them in each hole, how many mice would go into each hole?

4x6=24

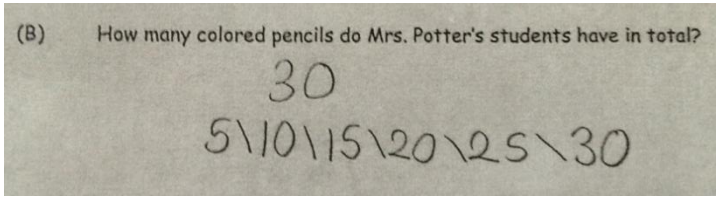
24÷6=4

24÷4=6

Post Assessment Results

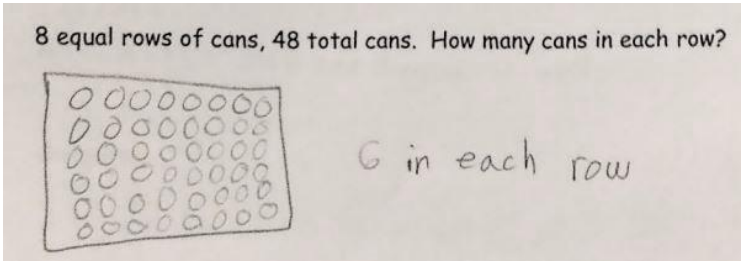
Reflecting on final interview and assessment, then comparing it to initial proficiency shown by students

- Jake- growth in *Conceptual Understanding* of relationship between operations
- Three students- *Procedural Fluency* in skip counting



Post Assessment Results

- Gabbie- growth in *Strategic Competence* shown through her models



- Earl- developed *Adaptive Reasoning* based on his ability to explain his thinking
- Jake- strength in *Adaptive Reasoning*, enjoys explaining his process
- Gabbie- weakness still with *Conceptual Understanding* of division but rise in *Productive Disposition* when approaching new types of problems

Reflection and Discussion

- Common Core Standards Reflection
  - Challenging standards
    - 3.OA.A.4
    - 3.OA.B.5
- Learning Progressions Reflection
  - Level 1 was reached and passed by most
  - Level 2 was reached for all
  - Level 3 proved harder to transition to

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