Developing Students' Understanding of Fraction Addition

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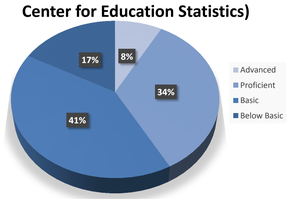
Introduction

On a recent National Assessment of Educational Progress (NAEP) test, less than half of fourth grade students performed at or above the "proficient" level (National Center for Education Statistics, 2013). Fractions, in particular, caused difficulties for students. For example:

- 45% of 4th graders were unable to add 3 fractions with like denominators (Fourth-Grade NAEP test item, block: 2013-4M3, Number 9)
- 26% of 4th graders were able to correctly add 3 fractions together with unlike denominators with a model (Fourth-Grade NAEP test item, block: 2013-4M7, Number 13)

Achievement-level Results in 4th Grade Mathematics (National Center for Education Statistics)



Purpose:

The Common Core State Standards (National Governors Association & Council of Chief State School Officers, 2010) require that students develop understanding of fraction equivalence in fourth grade. Fourth-graders are also to add and subtract fractions with like denominators. In our study, we aimed to study how a group of students finishing fourth grade might begin to understand addition and subtraction of fractions with unlike denominators.

Research Question: How can students' mathematical proficiency be developed for understanding addition and subtraction of fractions with unlike denominators?

References

National Center for Education Statistics. (2013). *A first look: 2013 mathematics and reading.* Retrieved from*http://nces.ed.gov/nationsreportcard/subject/publications/main2013/pdf/2014451.pdf*

National Governors Association for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Retrieved from http://www.corestandards.org/.

Theoretical framework: Attaining Proficiency

Ability to formulate, represent, and solve mathematical problems

Comprehension of mathematical concepts, operations, and relations

Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Conceptual Understanding

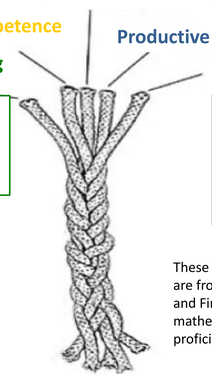
Strategic Competence

Productive Disposition

Adaptive Reasoning

Procedural Fluency

capacity for logical thought, reflection, explanation, and justification



Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

These five intertwined strands are from Kilpatrick, Swafford, and Findell's (2001) model of mathematical proficiency(pp.116-117).

Cramer, Wyberg, and Leavitt suggested that "concrete models are critical forms of representation and are needed to support students' understanding of, and operations with, fractions." (p. 490). So while providing manipulatives for the students to work with we also included contexts that the students could use to "visualize what the fractions represent (Tobias 2014, p. 378).

References

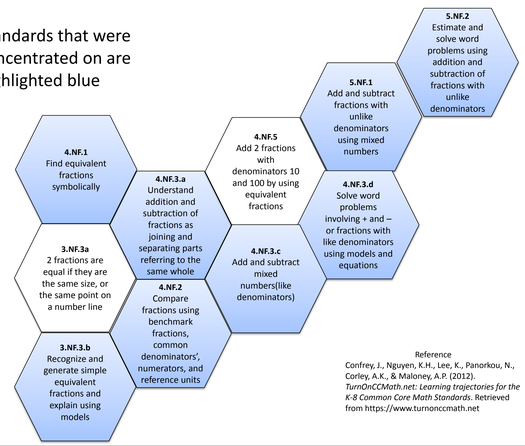
Cramer, K., Wyberg, T., & Leavitt, S. (2008). The role of representations in fraction addition and subtraction. *Mathematics Teaching in the Middle School, 13*(8), 490-496.

Kilpatrick, J., Swafford, J., Findell, B.(Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Tobias, J. M. (2014). Mixing strategies to compare fractions. *Mathematics Teaching in the Middle School, 19*(9), 376-381.

Theoretical framework: Conjectured learning progression(Confrey et al.2012)

Standards that were concentrated on are highlighted blue



Reference

Confrey, J., Nguyen, K.H., Lee, K., Panorkou, N., Corley, A.K., & Maloney, A.P. (2012). *TurnOnCCMath.net: Learning trajectories for the K-8 Common Core Math Standards*. Retrieved from https://www.turnoncmath.net

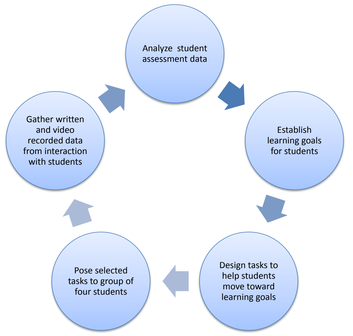
Methodology

Participants and procedure

Over the course of nine weeks, we worked with four students, two male and two female. We refer to them by the pseudonyms Steven, Sam, Lauren, and Vanessa. The four students were in the process of completing fourth grade and advancing into fifth. We designed and taught seven one-hour instructional sessions. All four students participated in all of the sessions. Before teaching the seven lessons, we conducted an individual 30-minute pre-assessment interview with each student to determine his or her knowledge of fractions. After teaching the seven lessons, we conducted individual 30-minute post-assessment interviews to discover what they had learned.

Methodology

PATHWAYS Instructional Cycle



Weekly Tasks for Each Cycle

- Teach and record one-hour session
- Collect students' written work and archive electronically
- Create a transcript of one-hour session from lesson video
- Code students' comments in each transcript using the 5 strands of mathematical proficiency
- Summarize students' progress along the learning progression, make conjectures about how to advance their thinking, and design classroom tasks accordingly

Data gathering and analysis:

- The pre-assessment interview questions were based on the Common Core learning progressions from our theoretical framework. During the post-assessment interviews eight weeks later, students were given the same set of items. Some key sample items were:

Four children each had a candy bar. The candy bars were identical.

- Ana ate $\frac{3}{8}$ of her candy bar.
- Brian ate $\frac{1}{2}$ of his candy bar.
- Carly ate $\frac{2}{9}$ of her candy bar.
- Don ate $\frac{2}{5}$ of his candy bar.

a. Who ate more than one half of the candy bar? Who ate less than one half? Explain your thinking.
b. Who ate the most? Who ate the least?
c. Put the children in order from who ate the most to who ate the least.

Len walks $\frac{3}{10}$ of a mile in the morning to school. He walks $\frac{2}{5}$ of a mile in the afternoon to a friend's house. Len says that he walks a total of $\frac{5}{15}$ of a mile total. Is his thinking reasonable?

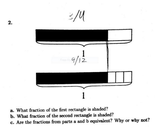
Diana works at a clothing store. She sold $\frac{1}{5}$ of the total number of green shirts on Monday and $\frac{3}{12}$ of the total number of green shirts on Tuesday. a. What fraction of green shirts did Diana sell all together on Monday and Tuesday? b. What fraction of green shirts did Diana not sell yet?

Sample items adapted from Illustrative Mathematics (http://www.illustrativemathematics.org) and PARCC (www.parcconline.org).

Empirical Teaching and Learning Trajectory:

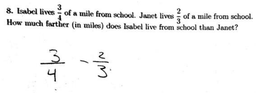
Initial Assessment Results

On the pre-assessment interview, students tended to have difficulty explaining why fractions were equivalent to one another. For example, Steven was given the following problem:



JS: Are the fractions from part A and B equivalent? Or similar? Are they the same?
Steven: There the same cause it's the same length. If you take the line it's the same length as that line. So, even though this one has like only has 4 and this one has 12 there the same.
JS: So they're the same length?
Steven: Yup

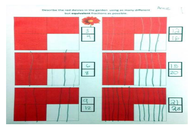
Students also had trouble adding and subtracting fractions with unlike denominators. For example, Vanessa was asked to respond to the item below:



KT: What do you need to do for this problem?
Vanessa: You need to change the denominator.
Student was unable to come up with a strategy and moved on from the question

Instructional Sessions(Weeks 2,4)

For our first lesson, week 2 of the program, the students were given an original fraction of ¾ and asked to make equivalent fractions. The following image depicts Lauren splitting her garden into equivalent fractions.



Conceptual Understanding Strengths

- Sam realized that the bigger the denominator, the smaller the fraction pieces were.
- Steven expressed that even though there were more pieces the fractions were still equivalent.

For our third lesson, week 4 of the program, the main focus was on finding common denominators with the fractions given by playing a game called one and out. During this time we were trying to address the weakness from week three. We wanted the students to understand fractions as being a single number instead of referring to it as two entities (numerator & denominator).

Strategic Competence Strength:
Students developed equivalent fractions so that they could find a common denominator in order to add a set of fractions together. While trying to get as close to one as possible, students accounted for both the numerator and denominator.

Instructional Sessions (Weeks 3,5)

The goal of instruction during week 3 and 5 was to expand on the students' fraction number sense through their ability to compare fractions to benchmarks such as ½ and a whole. Week 3's task focused on a pizza eating competition in which students were asked to order the contestants from who ate the least to who ate the most.

Strategic Competence Strength:

- Sam was able to make an equivalent fraction with an even numerator that made it easier to recognize the ½ benchmark
- To compare the fractions 7/8 and 20/24 students made the equivalent fraction 21/24 so that the set of fractions had the same common denominator

Conceptual Understanding Weakness:
Students, particularly Lauren and Sam, continued to refer to the pattern they noticed that the bigger the denominator, the smaller the pieces. When comparing several fractions they continued to reference this while ignoring the role of the numerator.

Week 5 included making estimations of the sum of two fractions. First students were asked to rely on their fraction number sense to determine whether or not the sum would be more or less than the ½ benchmark. Then students determined an estimation of the total using fraction bars as manipulatives.

Adaptive Reasoning Strength:
Students used the ½ benchmark to determine estimations for the total, determining whether the sum of the fraction was more or less.

Instructional Sessions (Weeks 6,7,8)

During weeks 6-8, we focused on addition and subtraction of fractions with unlike denominators and also included a few addition problems with fractions that had like denominators.

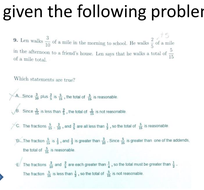
Strategic Competence Strength:
Vanessa and Lauren realized that to find a common denominator, you could multiply the denominators in the original fraction

Conceptual Understanding Strengths:
Vanessa and Sam were able to determine that making a common denominator makes the pieces the same size and that is why you need a common denominator

- During Week 6 students made the connection between the common denominator and size of the pieces which was a continuation of the lesson previously taught during week 5.
- During week 7, students were given an open ended problem. They were given a scenario that provoked them to come up with their own questions referring to the total amount of pizza left.
- During week 8 students played a game called "bump". They were asked to roll 2 die and the total that the die added up to was the problem they were suppose to solve. Each student would solve the addition or subtraction problem as individuals. Below is an example of the Bump Game:

Post-Assessment Results

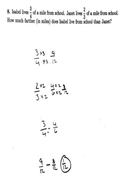
On the post-assessment interview, students were asked to explain if the answer of 5/15 was reasonable. For example, Steven was given the following problem:



Conceptual Understanding Strengths:
Steven knew that fractions with unlike denominators could not be added across.

Adaptive Reasoning Strength:
Steven recognized that all of the fractions were less than 1/2, but when added together the sum of them would be greater than a half.

Students were also given a problem that involved adding and subtracting fractions with unlike denominators. For example, Vanessa was asked to respond to the item below:



KT: OK. So if they need the same denominator what can you do to them?
Vanessa: You can – Oh. OK! You can divide this by 2.
KT: Which one are you dividing by 2?
Vanessa: ¾. *Student tried a different approach next*
Vanessa: I am going to multiply 4 X 3 to get 12. 12 – 9. So 9/12 and then I can make this times this by 2.
KT: Which one are you X by 2?
Vanessa: 3 – 2/3 and I made it into 4/6 so I am going to multiply that by 2.
KT: OK.
Vanessa: To get 12. And 8. So now I can subtract them.

Reflection and discussion:

Based on our research during this project we saw that the students had trouble with the CCSSM Standard 5.NF.2: estimate and solve word problems using addition and subtraction of fractions with unlike denominators. Students struggled to understand the meaning behind finding a common denominator; therefore the difficult points in the learning progression were the ones that addressed common denominators. Although they knew that you cannot add fractions straight across when the denominators are different they didn't know how to actually find a denominator that could be used between fractions. Since students were very familiar with making equivalent fractions and drawing models to represent their understanding this was our gateway into addition and subtraction with unlike denominators. The key helpful tools to move the students beyond creating equivalent fractions and onto finding common denominators were the fraction bars and virtual manipulatives. Both manipulatives helped the students to reinforce how to make equivalent fractions and visualize why finding a common denominator was necessary to perform the operations. Students needed to make the connection that the "pieces" of the whole had to be made the same size in order to add or subtract the fractions. In one lesson to obtain this we provided the students with fraction bars to measure out the sum of their fractions. This was a vital part of our learning progression. We find this standard significant in terms of requiring students to visually represent their ability to find common denominators for a set of fractions so that they are able to grasp this concept entirely.