MATH 160 Fall 2015 Exam III SCORE (on next page)

Date_____ Name_____

Math 160

APPLIED CALCULUS

EXAM III

Show all work.

Do not begin until told to do so.

I. Product Rule:

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + \frac{d}{dx}[g(x)] \cdot f(x).$$

$$(f \cdot g)' = f' \cdot g + g' \cdot f$$

II. Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)] \cdot g(x) - \frac{d}{dx}[g(x)] \cdot f(x)}{[g(x)]^2}.$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

SCORE_____

1. Given the function $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 6$:

   (a) Find the critical numbers.

   [4 points]

   (b) Find the intervals of increase and decrease.

   [5 points]

   (c) Find all relative extrema.

   [5 points]

   (d) Find the largest open interval(s) on which $f(x)$ is concave up and those on which $f(x)$ is concave down.

   [5 points]

   (e) Find all inflection points.

   [5 points]

2. Sketch a graph of the function defined in problem #1. Label all relative extrema and inflection points (if any exist).

[6 points]

3. Find the relative extrema of $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 10$ using the second derivative test (where possible).

[7 points]

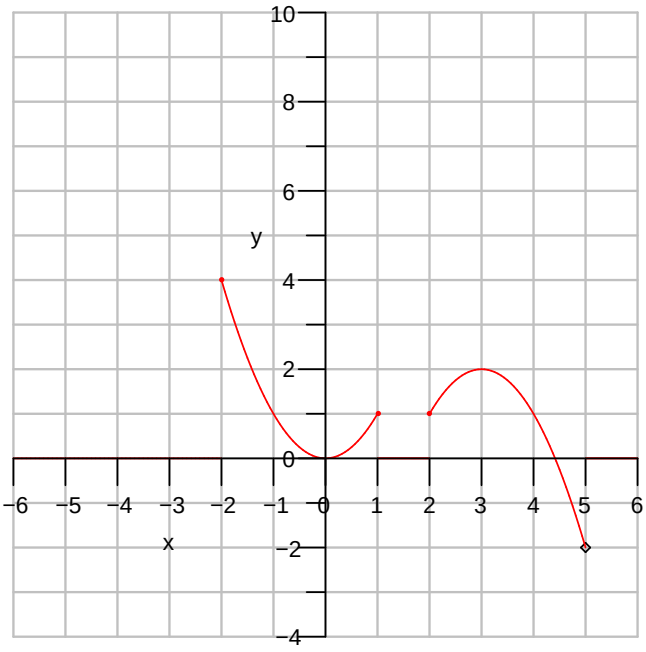
4. Find the elasticity of demand E at the given points and determine whether demand is inelastic, elastic or unitary. $\left(\text{Recall: } E(p) = -\dfrac{pf'(p)}{f(p)}\right)$

$$x = 10 + \frac{1}{p}; \quad p = 1$$

[7 points]

5. Consider the following graph. State the (a) domain, (b) range, (c) vertical intercept (if any),(d) concavity, (e) point(s) of inflection (if any)

[8 points]



6. Find the absolute extrema for $f(x) = x^3 - 3x^2 + 2$ on the closed interval $[1, 4]$.

[7 points]

7. The average pollen count in New York City on day x of the pollen season is $P(x) = 8x - 0.2x^2$ (for $0 \leq x \leq 40$). On which day is the pollen count highest?

[7 points]

8. The total weekly cost in dollars incurred by Herald Media Corp. in producing x DVDs is given by the total cost function

$$C(x) = 2500 + 2.2x \quad (0 \leq x \leq 8000)$$

Find the marginal cost and average marginal cost when x = 1000 and interpret your results.

[7 points]

9. For a Toyolla Terrain SUV,

$$M(x) = -0.015x^2 + 1.31x - 7.3,\ 30 \leq x \leq 60$$

represents the miles per gallon obtained at a speed of x mph. Find the absolute maximum miles per gallon and the absolute minimum, and the speeds at which they occur.

[6 points]

10. Esther's Desi Vegan Restaurant has been very busy recently. The demand for *chana masala*, one of its most popular dishes is $p = -\frac{1}{2}x + 33$, where p is the price based on the quantity x demanded. What should Esther charge to maximize revenue for this dish?

[8 points]

11. Find the fourth derivative, $f^{(4)}(x)$, of the given function:

$$f(x) = 2x^6 + 2x^4 + 3x^2 - 2$$

[6 points]

12. The marketing department of Telecon has determined that the demand for their smartphones obeys the relationship

$$p = -\frac{1}{2}x + 33. \quad (0 \leq x \leq 30,000)$$

where p denotes the phone's unit price (in dollars) and x denotes the quantity demanded.

(a) Find the Revenue function $R(x)$.

[3 points]

(b) Compute $R'(10,000)$ and intepret your result.

[4 points]