MATH 160 Fall 2015 Exam II SCORE (on next page)

Date__________________________ Name_____________________________________

Math 160

APPLIED CALCULUS

EXAM II

Show all work.

Do not begin until told to do so.

I. Product Rule:

$$\begin{aligned}\frac{d}{dx}[f(x)\cdot g(x)] &= \tfrac{d}{dx}[f(x)]\cdot g(x) + \tfrac{d}{dx}[g(x)]\cdot f(x) \\ &= f'\cdot g + g'\cdot f\end{aligned}$$

II. Quotient Rule:

$$\begin{aligned}\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] &= \frac{\tfrac{d}{dx}[f(x)]\cdot g(x) - \tfrac{d}{dx}[g(x)]\cdot f(x)}{[g(x)]^2} \\ &= \frac{f'\cdot g - g'\cdot f}{g^2}\end{aligned}$$

III. Chain Rule:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\cdot g'(x)$$

SCORE______________

1. Find the slope of the tangent line to $f(x) = x^3 + 2x + 1$ at $x = 1$.

[8 points]

Solution:

The slope of the tangent at $x = 1$ is equivalent to the derivative at $x = 1$, $f'(1)$, $f'(x) = 3x^2 + 2$, so $f'(1) = 3(1)^2 + 2 = 5$. The slope of the tangent line at $x = 1$ is 5.

2. Let $f(x) = -2x^2 - 3$:

(a) Find the average rate of change over the interval [1, 3].

[6 points]

(b) Find the instantaneous rate of change at x = 3.

[6 points]

3. Using the limit definition, find the derivative of $f(x) = 4x^2$.

[7 points]

Solution:

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0} \frac{4(x+h)^2 - 4x^2}{h} \\
&= \lim_{h\to 0} \frac{4(x^2+2xh+h^2) - 4x^2}{h} \\
&= \lim_{h\to 0} \frac{4x^2+8xh+4h^2-4x^2}{h} \\
&= \lim_{h\to 0} \frac{8xh+4h^2}{h} \\
&= \lim_{h\to 0} \frac{h(8x+4h)}{h} \\
&= \lim_{h\to 0} 8x+4h \\
&= 8x+4(0) = 8x
\end{aligned}$$

4. Find the equation of the tangent line to $f(x) = 5x^2 - 4x - 12$ at the point $(-2, f(-2))$. (*Remember the slope of the tangent line at $x = a$ is $f'(a)$*).

[8 points]

5. Given $f(x) = \dfrac{5}{x} + 8x^2 + 4\sqrt{x}$, find $f'(1)$.

[8 points]

Solution:

$$\begin{aligned} f(x) &= 5x^{-1} + 8x^2 + 4x^{1/2} \\ \text{so } f'(x) &= -5x^{-2} + 16x + 2x^{-1/2} \\ f'(x) &= \frac{-5}{x^2} + 16x + \frac{2}{\sqrt{x}} \\ f'(1) &= \frac{-5}{1^2} + 16(1) + \frac{2}{\sqrt{1}} \\ &= -5 + 16 + 2 = 13 \end{aligned}$$

6. Find the derivative of $f(x) = \pi^3$.

[6 points]

7. Find the derivative of $f(x) = 7x^{-3} + 3x^{5/3}$.

[6 points]

Solution:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[7x^{-3} + 3x^{5/3}\right] \\ &= -21x^{-4} + 5x^{2/3} \end{aligned}$$

8. Find the derivative of $f(x) = 5x^6 + 3x^2 - \frac{2}{x}$.

[6 points]

9. Find the derivative of the function $f(x) = \frac{2x^8}{6x^2}$ in two ways:

(a) Using Quotient rule.

[6 points]

Solution:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[\frac{2x^8}{6x^2}\right] \\ &= \frac{\frac{d}{dx}\left[2x^8\right](6x^2) - \frac{d}{dx}\left[6x^2\right](2x^8)}{(6x^2)^2} \\ &= \frac{(16x^7)(6x^2) - (12x)(2x^8)}{(6x^2)^2} \\ &= \frac{96x^9 - 24x^9}{36x^4} \\ &= \frac{72x^9}{36x^4} = 2x^5 \end{aligned}$$

(b) Simplifying the function and using the Power rule.

[6 points]

Solution:

$$\begin{aligned} f(x) &= \frac{2x^8}{6x^2} \\ &= \frac{x^6}{3} = \frac{1}{3}x^6 \\ f'(x) &= \frac{1}{3} \cdot 6x^5 \\ &= 2x^5 \end{aligned}$$

10. Find functions f and g such that the given function is $f(g(x))$.

$$\sqrt{3x^4 - 7x^2 + 5}$$

[6 points]

Solution:

f is the "outer" function and g is the "inner" function,
So: $y = f(u) = \sqrt{u} = u^{1/2}$, and $u = g(x) = 3x^4 - 7x^2 + 5$,
in terms of x only, we have $f(x) = x^{1/2}$ and $g(x) = 3x^4 - 7x^2 + 5$.

11. Find $x(x^2 - 1)^4$ using product rule (*Hint: Remember you must use chain rule as well*).

[7 points]

12. Calculate the derivative of $f(x) = \dfrac{x^2 - 2x + 3}{x + 1}$.

[7 points]

Solution:

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}\left[\frac{x^2 - 2x + 3}{x + 1}\right] \\
&= \frac{\frac{d}{dx}\left[x^2 - 2x + 3\right](x+1) - \frac{d}{dx}\left[x+1\right](x^2 - 2x + 3)}{(x+1)^2} \\
&= \frac{(2x-2)(x+1) - 1(x^2 - 2x + 3)}{(x+1)^2} \\
&= \frac{2x^2 - 2 - x^2 + 2x - 3}{(x+1)^2} \\
&= \frac{x^2 + 2x - 5}{(x+1)^2}
\end{aligned}
$$

13. Find the derivative of the function in question #10.

[7 points]

Solution:

$$
\begin{aligned}
\frac{d}{dx}\left[\sqrt{3x^4 - 7x^2 + 5}\right] &= \frac{d}{dx}\left[(3x^4 - 7x^2 + 5)^{1/2}\right] \\
&= \frac{1}{2}(3x^4 - 7x^2 + 5)^{-1/2} \cdot \frac{d}{dx}\left[3x^4 - 7x^2 + 5\right] \\
&= \frac{1}{2}(3x^4 - 7x^2 + 5)^{-1/2} \cdot (12x^3 - 14x) \\
&= (6x^3 - 7x)(3x^4 - 7x^2 + 5)^{-1/2} \\
&= \frac{6x^3 - 7x}{\sqrt{3x^4 - 7x^2 + 5}}
\end{aligned}
$$