MATH 160 Fall 2015 Exam III SCORE (on next page)

Date_____Name_____

Math 160 APPLIED CALCULUS EXAM III

Show all work. Do not begin until told to do so.

I. Product Rule:

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + \frac{d}{dx}[g(x)] \cdot f(x).$$
$$(f \cdot g)' = f' \cdot g + g' \cdot f$$

II. Quotient Rule:

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{\frac{d}{dx} [f(x)] \cdot g(x) - \frac{d}{dx} [g(x)] \cdot f(x)}{[g(x)]^2}.$$
$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$$

SCORE_____

1. Given the function
$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 6$$
:

(a) Find the critical numbers.

[4 points]

(b) Find the intervals of increase and decrease.

[5 points]

(c) Find all relative extrema.

[5 points]

(d) Find the largest open interval(s) on which f(x) is concave up and those on which f(x) is concave down.

[5 points]

Solution: Find where f''(x) = 0, f''(x) = 2x - 2 2x - 2 = 0 2x = 2x = 1, possible inflection point

Test intervals are $(-\infty, 1)$ and $(1, \infty)$. $(-\infty, 1)$: x = 0 f''(0) = 2(0) - 2 = -2, concave downward on this interval. $(1, \infty)$: x = 2, f''(2) = 2(2) - 2 = 2, concave upward on this interval.

(e) Find all inflection points.

[5 points]

Solution:

There is an inflection point at x = 1 since concavity changes there, the inflection point is $\left(1, \frac{7}{3}\right)$

2. Sketch a graph of the function defined in problem #1. Label all relative extrema and inflection points (if any exist).

[6 points]

3. Find the relative extrema of $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 10$ using the second derivative test (where possible).

[7 points]

Solution:

Find where f'(x) = 0,

$$f'(x) = 0x^2 - 4x - 5 = 0(x+1)(x-5) = 0$$

x = 5, x = -1 are critical numbers.

f''(x) = 2x - 4f''(5) = 2(5) - 4 = 6, concave upward, so there is a relative minimum at x = 5. f''(-1) = 2(-1) - 4 = -6, concave downward, so there is a relative maximum at x = -1.

relative maximum, $\left(-1, -\frac{22}{3}\right)$; relative minimum, $\left(5, -\frac{130}{3}\right)$

4. Find the elasticity of demand E at the given points and determine whether demand is inelastic, elastic or unitary. (Recall: $E(p) = -\frac{pf'(p)}{f(p)}$) $x = 10 + \frac{1}{p}; \ p = 1$ [7 points]

Solution:

$$f(p) = 10 + \frac{1}{p}$$

$$f'(p) = -\frac{1}{p^2}$$

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p\left(-\frac{1}{p^2}\right)}{10 + \frac{1}{p}} = \frac{\frac{1}{p}}{10 + \frac{1}{p}}$$

$$E(1) = \frac{\frac{1}{1}}{10 + \frac{1}{1}} = \frac{1}{11} < 1, \text{ demand is inelastic}$$

5. Consider the following graph. State the (a) domain, (b) range, (c) vertical intercept (if any),(d) concavity, (e) point(s) of inflection (if any)

[8 points]



6. Find the absolute extrema for $f(x) = x^3 - 3x^2 + 2$ on the closed interval [1,4].

[7 points]

Solution:

Find critical numbers:

$$f'(x) = 0
 3x^2 - 6x = 0
 3x(x-2) = 0$$

x = 0, x = 2 are critical numbers.

x = 0 not in interval, we exclude from our calculations.

$$\begin{split} f(1) &= 1^3 - 3(1)^2 + 2 = 0 \\ f(2) &= 2^3 - 3(2)^2 + 2 = -2 \\ f(4) &= 4^3 - 3(4)^2 + 2 = 18. \end{split}$$

absolute maximum, (4, 18)absolute minimum, (2, -2)

7. The average pollen count in New York City on day x of the pollen season is $P(x) = 8x - 0.2x^2$ (for $0 \le x \le 40$). On which day is the pollen count highest?

[7 points]

8. The total weekly cost in dollars incurred by Herald Media Corp. in producing x DVDs is given by the total cost function

$$C(x) = 2500 + 2.2x \quad (0 \le x \le 8000)$$

Find the marginal cost and average marginal cost when x = 1000 and interpret your results.

[7 points]

Solution:

C'(x) = 2.2C'(1000) = 2.2

The approximate cost of producing the 1001st DVD is \$2.20.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{2500 + 2.2x}{x} = \frac{2500}{x} + 2.2$$
$$\overline{C}'(x) = -\frac{2500}{x^2}$$
$$\overline{C}'(1000) = -\frac{2500}{1000^2} = \frac{2500}{1000000} = 0.0025$$

The average cost per DVD is increasing by 0.25 cents when the 1000th DVD is produced.

9. For a Toyolla Terrain SUV,

$$M(x) = -0.015x^2 + 1.31x - 7.3, \ 30 \le x \le 60$$

represents the miles per gallon obtained at a speed of x mph. Find the absolute maximum miles per gallon and the absolute minimum, and the speeds at which they occur.

[6 points]

10. Esther's Desi Vegan Restaurant has been very busy recently. The demand for *chana masala*, one of its most popular dishes is $p = -\frac{1}{2}x + 33$, where p is the price based on the quantity x demanded. What should Esther charge to maximize revenue for this dish?

[8 points]

Solution:

We first find our Revenue function, $R(x) = px = (-\frac{1}{2}x + 33)x = -\frac{1}{2}x^2 + 33x$. We next find our critical numbers, where R'(x) = 0, R'(x) = -x + 33.

> -x + 33 = 0 -x = -33x = 33, critical number

The domain of R(x) is $0 \le x \le 66$, which is based on the demand equation.

This is an absolute extrema problem.

$$\begin{split} R(0) &= -\frac{1}{2}(0)^2 + 33(0) = 0\\ R(33) &= -\frac{1}{2}(33)^2 + 33(33) = 1633.5\\ R(66) &= -\frac{1}{2}(66)^2 + 33(66) = 0. \end{split}$$

Revenue is maximized when x = 33, Esther should charge $p = -\frac{1}{2}(33) + 33 = 16.5$, \$16.50.

11. Find the fourth derivative, $f^{(4)}(x)$, of the given function:

$$f(x) = 2x^6 + 2x^4 + 3x^2 - 2$$

[6 points]

12. The marketing department of Telecon has determined that the demand for their smartphones obeys the relationship

$$p = -\frac{1}{2}x + 33. \quad (0 \le x \le 30,000)$$

where p denotes the phone's unit price (in dollars) and x denotes the quantity demanded.

(a) Find the Revenue function R(x).

[3 points]

Solution: $R(x) = px = (-\frac{1}{2}x + 33)x = -\frac{1}{2}x^2 + 33x$

(b) Compute R'(10,000) and integret your result.

[4 points]

Solution: R'(x) = -x + 33R'(10000) = -10000 + 33 = -9967

The revenue from selling the 10,001st unit is approximately -\$9967.

Note: This problem's demand equation is actually wrong, it should be p = -0.03x + 1200.