SCORE_____

1. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, evaluate each expression and find their domains:

(a) $(f + g)(x)$

[3 points]

(b) $(f - g)(x)$

[3 points]

(c) $(f \cdot g)(x)$

[3 points]

(d) $(f/g)(x)$

[3 points]

(e) $(f \circ g)(x)$

[3 points]

2. Find the inverse of $f(x)$ in the previous problem and its domain.

[5 points]

3. Evaluate the following:

$$\lim_{x \to -1} (2x^3 - 12x - 4)$$

[5 points]

4. Evaluate the following:

$$\lim_{x \to 5} \frac{x^2 - 2x - 15}{x - 5}$$

[5 points]

5. Given the function:

$$f(x) = \begin{cases} 2 - x & \text{when } x < -1 \\ x & \text{when } -1 \leq x < 1 \\ (x-1)^2 & \text{when } x \geq 1 \end{cases}$$

find the following limits (if they exist).

(a) $\lim_{x \to -1^-} f(x)$

[5 points]

(b) $\lim_{x \to -1^+} f(x)$

[5 points]

(c) $\lim_{x \to -1} f(x)$

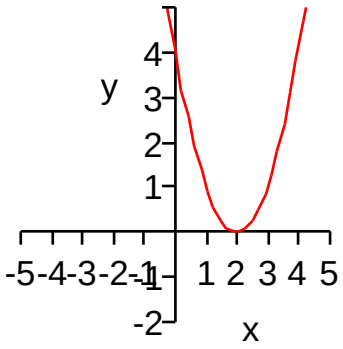
[5 points]

6. Give the formal definition of a limit.

[5 points]

7. The graph of f(x) is given. Draw the graph of f(x - 2) - 1.

[5 points]



8. Using the limit definition, find the derivative of $f(x) = x^2 + 2x + 3$.

[5 points]

9. Given the function f(x),

x	f(x)
0.5	2.5
0.9	8.1
0.99	9.8
1	10
1.01	10.2
1.1	12.1
1.5	22.5

Estimate the slope of the tangent line at $x = 1$ by calculating the slopes of the secant lines over the following intervals:

$$(0.5, 1), (0.9, 1), (0.99, 1), (1, 1.01), (1, 1.1), (1, 1.5)$$

[10 points]

10. Find an equation of the tangent to $f(x) = x^4 - x$ at the point (-1, 2).

[10 points]

11. Determine the vertical and horizontal aysmptotes (if any exist) of the following function:

$$f(x) = \frac{x^2 + 4}{x^2 - 1}$$

[8 points]

12. Find the derivative of the function $f(x) = 3x(2x + 5)^6$.

[7 points]

13. Find the derivative of the function $f(x) = 5x^4 - \ln(2x^2) + xe^x$.

[5 points]

14. Find $\dfrac{dy}{dx}$ by implicit differentiation.

$$xe^y - 10x + 3y = 0$$

[5 points]

15.A companys profit from selling x units of an item is $P = 1000x\frac{1}{2}x^2$ dollars. If sales are growing at a rate of 20 per day, find how rapidly profit is growing (in dollars per day) when 600 units have been sold.

[5 points]

16. Find the absolute extrema of the function on the closed interval $[-1, 2]$.

$$g(x) = 3x^4 - 4x^3$$

[8 points]

17. Find all numbers c that satisfy the conclusion of the Mean Value Theorem on the interval $[0, 2]$

$$f(x) = x^3 + x - 1$$

[6 points]

18. Let $V(t) = 16e^{-0.5t}$ be the amount of coffee (in ounces) left in Star's cup t minutes after she receives her drink. Find and interpret $V'(2)$.

[6 points]

19. Let $f(x) = -2x^3 + 4x + 3$.

(a) Find all relative extrema.

[5 points]

(b) Find the open interval(s) on which $f(x)$ is increasing and/or decreasing.

[5 points]

(c) Find the largest open interval(s) on which $f(x)$ is concave up and those on which $f(x)$ is concave down.

[5 points]

(d) Find all inflection points.

[5 points]

20. Use L'Hôpital's Rule to evaluate the following limits. If L'Hôpital's Rule does not apply, explain why.

(a) $\lim_{t \to 0} \dfrac{e^{3t} - 1}{t}$

[5 points]

(b) $\lim_{t \to \infty} \dfrac{x}{\sqrt{x^2 + 1}}$

[5 points]

21. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

[10 points]

22. The average pollen count in New York City on day x of the pollen season is $P(x) = 8x - 0.2x^2$ (for $0 \leq x \leq 40$). On which day is the pollen count highest?

[6 points]

23. Find the linearization, that is, the tangent line approximation of the function $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.99}$ and $\sqrt{4.03}$.

[8 points]

24. Find the most general antiderivative of the following functions.

(a) $f(x) = 1 - x^3 + 5x^5 - 3x^7$

[5 points]

(b) $f(x) = 2x + 3x^{1.7}$

[5 points]

25. Find the antiderivative f of f' that satisfies the given condition.

(a) $f'(x) = 8x^3 + 12x + 3,\ f(1) = 6$

[5 points]

(b) $f'(x) = \sqrt{x}(6 + 5x),\ f(1) = 10$

[5 points]