SCORE______________

1. Find the domain of $f(x) = \dfrac{1}{x-4}$.

[8 points]

Solution:

f is undefined when x -4 = 0, So
$x - 4 \neq 0$
$x \neq 4$
$D : \{x \,|\, x \neq 4\}$

2. Find the inverse of $f(x)$ in the previous problem and its domain.

[8 points]

Solution:

$$\begin{aligned}
f(x) &= \frac{1}{x-4} \\
y &= \frac{1}{x-4} \\
x &= \frac{1}{y-4} \\
x(y-4) &= 1 \\
(y-4) &= \frac{1}{x} \\
y &= \frac{1}{x} + 4 \\
f^{-1}(x) &= \frac{1}{x} + 4 \\
f^{-1}(x) &\rightarrow D : \{x \,|\, x \neq 0\} \;\; R : \{y \,|\, y \neq 4\} \\
f(x) &\rightarrow D : \{x \,|\, x \neq 4\} \;\; R : \{y \,|\, y \neq 0\}
\end{aligned}$$

3. Let $f(x) = \sqrt{x-3}$ and $g(x) = x+4$. Find each of the following and their domains:

(a) $(f+g)(x)$

[3 points]

Solution:
$(f+g)(x) = f(x) + g(x) = \sqrt{x-3} + x + 4$

Domain: $D_{f+g} : [3, \infty) = \{x | x \geq 3\}$

(b) $(f-g)(x)$

[3 points]

Solution:
$(f-g)(x) = f(x) - g(x) = \sqrt{x-3} - (x+4) = \sqrt{x-3} - x - 4$

Domain: $D_{f-g} : [3, \infty) = \{x | x \geq 3\}$

(c) $(f \cdot g)(x)$

[3 points]

Solution:
$(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{x-3} \cdot (x+4)$

Domain: $D_{f \cdot g} : [3, \infty) = \{x | x \geq 3\}$

(d) $(f/g)(x)$

[3 points]

Solution:
$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x-3}}{(x+4)}$

Domain: $D_{f/g} : [3, \infty) = \{x | x \geq 3\}$

(e) $(f \circ g)(x)$

[3 points]

Solution:
$(f \circ g)(x) = f(g(x)) = \sqrt{(x+4) - 3} = \sqrt{x+1}$

Domain: $D_{f \circ g} : [-1, \infty) = \{x | x \geq -1\}$

4. Show that $f(x) = 3x + 2$ is one-to-one either graphically or algebraically.

[7 points]

Solution:

$f(x_1) = f(x_2)$
$3x_1 + 2 = 3x_2 + 2$
$3x_1 = 3x_2$ *Substract 2 from both sides*
$x_1 = x_2$ *Divide both sides by 3*
Thus f is one-to-one

5. Do the following.

(a) Express the following function in the form $f \circ g \circ h$:

$$(\sqrt{x^3} + 1)^4$$

[4 points]

(b) Find $f \circ g \circ h$:

$$f(x) = 2x - 3, \quad g(x) = x^2, \quad h(x) = \sqrt{x+3}$$

[4 points]

Solution:

$$\begin{aligned} f \circ g \circ h &= f(g(h(x))) \\ &= f(g(\sqrt{x+3})) \\ &= f(\sqrt{x+3}^2) \\ &= f(x+3) \\ &= 2(x+3) - 3 \\ &= 2x + 6 - 3 \\ &= 2x + 3 \end{aligned}$$

6. Give the definition of a function f in terms of sets. Define range.

[7 points]

Solution:

A function $f : A \to B$ is a rule that assigns every value in the set A to exactly one value in the set B. The range of f is the set of all $f(x)$'s for each $x \in A$, that is, $\{f(x)|x \in A\}$

7. Solve the inequality for x.

$$2 \ln x \leq 6$$

[7 points]

8. Find x.

$$e^{2-3x} = 1$$

[7 points]

Solution:

$$\begin{aligned} e^{2-3x} &= 1 \\ \ln e^{2-3x} &= \ln 1 \\ 2 - 3x &= 0 \\ 2 &= 3x \\ \frac{2}{3} &= x \end{aligned}$$

9. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 mi and in June it cost her \$460 to drive 800 mi.

(a) Express the monthly cost C as a function of the distance driven d, assume a linear relationship.

[4 points]

(b) Using part (a), predict the cost of driving 1500 miles per month.

[4 points]

10. Given $f(x) = 6x^2 + 1$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$ (Assume $h \neq 0$.)

[7 points]

Solution:

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{6(x+h)^2 + 1 - (6x^2 + 1)}{h} \\
&= \frac{6(x^2 + 2xh + h^2) + 1 - 6x^2 - 1}{h} \\
&= \frac{6x^2 + 12xh + 6h^2 + 1 - 6x^2 - 1}{h} \\
&= \frac{12xh + 6h^2}{h} \\
&= \frac{h(12x + 6h)}{h} \\
&= 12x + 6h
\end{aligned}$$

11.Consider the function $f(x) = 3 - x^2$ and the point P(1, 2) on the graph f.

(a) Graph f and the secant lines passing through P(1, 2) and Q(x, f(x)) for x-values 2 and 1.5.

[5 points]

(b) Find the slope of each secant line.

[5 points]

12. The graph of f(x) is given. Draw the graph of f(x - 3) + 1.

[6 points]