

SCORE\_\_\_\_\_

1. Use the Squeeze theorem to find the  $\lim_{x \rightarrow 0} f(x)$  if

$$3 - x^3 \leq f(x) \leq 3 + x^3.$$

[5 points]

2. Evaluate the following

$$\lim_{x \rightarrow 2} \frac{x + 2}{x^2 + 3x + 5}.$$

[5 points]

3. Give the formal definition of a limit.

[5 points]

4. Find a suitable  $\delta$  which proves that

$$\lim_{x \rightarrow -3} (9 - 2x) = 15$$

[6 points]

5. Find the following limit  $L$ . Then find  $\delta > 0$  such that  $|f(x) - L| < 0.01$  whenever  $0 < |x - a| < \delta$ .

$$\lim_{x \rightarrow 2} (2x + 8)$$

[6 points]

6. Find the x-values (if any) at which  $f$  is not continuous.

$$f(x) = \frac{x + 1}{x^2 - 4x + 3}$$

[8 points]

7. Use the Intermediate Value Theorem to show that there is a  $c$ ,  $a \leq c \leq b$ , such that  $f(c) = 9$  on the interval  $[0, 5]$ .

$$f(x) = x^2 + x - 1$$

[8 points]

8. Evaluate the following limit

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 2}}{3x + 1}.$$

[8 points]

9. Determine the vertical and horizontal asymptotes (if any exist) of the following function

$$f(x) = \frac{3x - 15}{x^2 - 25}.$$

[10 points]

10. Using the limit definition, find the derivative of  $f(x) = x^2 + x - 1$ .

[8 points]

11. Using the limit definition, find the derivative of  $f(x) = \frac{2}{x+1}$ .

[8 points]

12. Find the derivative of  $f(x) = 2x^4 - 3x^2 + \pi x - 18$ .

[7 points]

13. Find the derivative of  $f(x) = 6x^5 + \frac{8}{x^2} - 3e^x$

[7 points]

Bonus. Do only one of the

- (a) Prove that  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$ , (b) Find a suitable  $\delta$  which proves  $\lim_{x \rightarrow a} \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{a}}$   
[6 points]