SCORE______________

1. Use the Squeeze theorem to find the $\lim_{x \to 0} f(x)$ if

$$3 - x^3 \leq f(x) \leq 3 + x^3.$$

[5 points]

Solution:

$$\begin{aligned} \text{Now, } \lim_{x \to 0} 3 - x^3 &= 3 \\ \& \lim_{x \to 0} 3 + x^3 &= 3 \\ \text{Therefore, } \lim_{x \to 0} f(x) &= 3 \end{aligned}$$

2. Evaluate the following

$$\lim_{x \to 2} \frac{x + 2}{x^2 + 3x + 5}.$$

[5 points]

3. Give the formal definition of a limit.

[5 points]

4. Find a suitable δ which proves that

$$\lim_{x \to -3} (9 - 2x) = 15$$

[6 points]

Solution:

$$\begin{aligned}
|(9-2x)-15| &\leq k|x+3| \\
\text{Work from the left-hand side} & \\
|(9-2x)-15| &= |-6-2x| \\
&= |(-2)(x+3)| \\
&= |(-2)||(x+3)| \\
&= 2|x+3| \\
\text{Thus, k} = 2 & \\
\text{Now } \delta = \epsilon/k, \text{ so } \delta = \epsilon/2. &
\end{aligned}$$

5. Find the following limit L. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - a| < \delta$.

$$\lim_{x \to 2}(2x + 8)$$

[6 points]

6. Find the x-values (if any) at which f is not continuous.

$$f(x) = \frac{x+1}{x^2 - 4x + 3}$$

[8 points]

7. Use the Intermediate Value Theorem to show that there is a c, $a \leq c \leq b$, such that $f(c) = 9$ on the interval $[0, 5]$.

$$f(x) = x^2 + x - 1$$

[8 points]

8. Evaluate the following limit

$$\lim_{x\to\infty} \frac{\sqrt{4x^2 - 2}}{3x + 1}.$$

[8 points]

9. Determine the vertical and horizontal asymptotes (if any exist) of the following function

$$f(x) = \frac{3x - 15}{x^2 - 25}.$$

[10 points]

Solution:

Vertical Asymptotes:

$$\begin{aligned} x^2 - 25 &= 0 \\ (x+5)(x-5) &= 0 \\ x &= \pm 5 \end{aligned}$$

$$\begin{aligned} \lim_{x \to 5} \frac{3x-15}{x^2-25} &= \lim_{x \to 5} \frac{3(x-5)}{(x-5)(x+5)} \\ &= \lim_{x \to 5} \frac{3}{(x+5)} \\ &= \tfrac{3}{10} \end{aligned}$$

$$\begin{aligned} \lim_{x \to -5^+} \frac{3x-15}{x^2-25} &= \lim_{x \to -5^+} \frac{3(x-5)}{(x-5)(x+5)} \\ &= \lim_{x \to -5^+} \frac{3}{(x+5)} \\ &= \infty \end{aligned}$$

Vertical Asymptote: x = -5

Horizontal Asymptotes:

$$\begin{aligned} \lim_{x \to \infty} \frac{3x-15}{x^2-25} &= \lim_{x \to \infty} \frac{3x-15}{x^2-25} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\ &= \lim_{x \to \infty} \frac{\frac{3}{x} - \frac{15}{x^2}}{1 - \frac{25}{x^2}} \\ &= \tfrac{0-0}{1-0} \\ &= 0 \end{aligned}$$

Horizontal Asymptote: y = 0

Note: $\displaystyle \lim_{x \to -\infty} \frac{3x-15}{x^2-25} = 0$ also.

10. Using the limit definition, find the derivative of $f(x) = x^2 + x - 1$.

[8 points]

11. Using the limit definition, find the derivative of $f(x) = \dfrac{2}{x+1}$.

[8 points]

Bonus. Do only one of the

(a) Prove that $\lim_{x \to 0} \dfrac{\cos x - 1}{x} = 0$, (b) Find a suitable δ which proves $\lim_{x \to a} \dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{a}}$

[6 points]