

SCORE.....

1. Use the Squeeze theorem to find the $\lim_{x \rightarrow 0} f(x)$ if

$$3 - x^3 \leq f(x) \leq 3 + x^3.$$

[5 points]

2. Evaluate the following

$$\lim_{x \rightarrow 2} \frac{x + 2}{x^2 + 3x + 5}.$$

[5 points]

3. Give the formal definition of a limit.

[5 points]

4. Find a suitable δ which proves that

$$\lim_{x \rightarrow -3} (9 - 2x) = 15$$

[6 points]

5. Find the following limit L . Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - a| < \delta$.

$$\lim_{x \rightarrow 2} (2x + 8)$$

[6 points]

6. Find the x-values (if any) at which f is not continuous.

$$f(x) = \frac{x + 1}{x^2 - 4x + 3}$$

[8 points]

7. Use the Intermediate Value Theorem to show that there is a c , $a \leq c \leq b$, such that $f(c) = 9$ on the interval $[0, 5]$.

$$f(x) = x^2 + x - 1$$

[8 points]

8. Evaluate the following limit

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 2}}{3x + 1}.$$

[8 points]

9. Determine the vertical and horizontal asymptotes (if any exist) of the following function

$$f(x) = \frac{3x - 15}{x^2 - 25}.$$

[10 points]

10. Using the limit definition, find the derivative of $f(x) = x^2 + x - 1$.

[8 points]

11. Using the limit definition, find the derivative of $f(x) = \frac{2}{x+1}$.

[8 points]

Bonus. Do only one of the

(a) Prove that $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$, (b) Find a suitable δ which proves $\lim_{x \rightarrow a} \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{a}}$
[6 points]