

SCORE_____

1. Use the Squeeze theorem to find the $\lim_{x \rightarrow 0} f(x)$ if

$$3 - x^3 \leq f(x) \leq 3 + x^3.$$

[5 points]

Solution:

$$\text{Now, } \lim_{x \rightarrow 0} 3 - x^3 = 3$$

$$\& \lim_{x \rightarrow 0} 3 + x^3 = 3$$

$$\text{Therefore, } \lim_{x \rightarrow 0} f(x) = 3$$

2. Evaluate the following

$$\lim_{x \rightarrow 2} \frac{x + 2}{x^2 + 3x + 5}.$$

[5 points]

3. Give the formal definition of a limit.

[5 points]

4. Find a suitable δ which proves that

$$\lim_{x \rightarrow -3} (9 - 2x) = 15$$

[6 points]

Solution:

$$\begin{aligned} |(9 - 2x) - 15| &\leq k|x + 3| \\ \text{Work from the left-hand side} \\ |(9 - 2x) - 15| &= |-6 - 2x| \\ &= |(-2)(x + 3)| \\ &= |(-2)||x + 3| \\ &= 2|x + 3| \end{aligned}$$

Thus, $k = 2$

Now $\delta = \epsilon/k$, so $\delta = \epsilon/2$.

5. Find the following limit L . Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - a| < \delta$.

$$\lim_{x \rightarrow 2} (2x + 8)$$

[6 points]

6. Find the x-values (if any) at which f is not continuous.

$$f(x) = \frac{x + 1}{x^2 - 4x + 3}$$

[8 points]

7. Use the Intermediate Value Theorem to show that there is a c , $a \leq c \leq b$, such that $f(c) = 9$ on the interval $[0, 5]$.

$$f(x) = x^2 + x - 1$$

[8 points]

8. Evaluate the following limit

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 2}}{3x + 1}.$$

[8 points]

9. Determine the vertical and horizontal asymptotes (if any exist) of the following function

$$f(x) = \frac{3x - 15}{x^2 - 25}.$$

[10 points]

Solution:

Vertical Asymptotes:

$$\begin{aligned} x^2 - 25 &= 0 \\ (x + 5)(x - 5) &= 0 \\ x &= \pm 5 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 5} \frac{3x - 15}{x^2 - 25} &= \lim_{x \rightarrow 5} \frac{3(x - 5)}{(x - 5)(x + 5)} \\ &= \lim_{x \rightarrow 5} \frac{3}{(x + 5)} \\ &= \frac{3}{10} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -5^+} \frac{3x - 15}{x^2 - 25} &= \lim_{x \rightarrow -5^+} \frac{3(x - 5)}{(x - 5)(x + 5)} \\ &= \lim_{x \rightarrow -5^+} \frac{3}{(x + 5)} \\ &= \infty \end{aligned}$$

Vertical Asymptote: $x = -5$

Horizontal Asymptotes:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x - 15}{x^2 - 25} &= \lim_{x \rightarrow \infty} \frac{3x - 15}{x^2 - 25} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{3}{x} - \frac{15}{x^2}}{1 - \frac{25}{x^2}} \\ &= \frac{0 - 0}{1 - 0} \\ &= 0 \end{aligned}$$

Horizontal Asymptote: $y = 0$

$$\text{Note: } \lim_{x \rightarrow -\infty} \frac{3x - 15}{x^2 - 25} = 0 \text{ also.}$$

10. Using the limit definition, find the derivative of $f(x) = x^2 + x - 1$.

[8 points]

11. Using the limit definition, find the derivative of $f(x) = \frac{2}{x+1}$.

[8 points]

Bonus. Do only one of the

(a) Prove that $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$, (b) Find a suitable δ which proves $\lim_{x \rightarrow a} \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{a}}$
[6 points]