1. Find functions f and g such that the given function is $f(g(x))$ and find the derivative.

$$(2x^2 - 7x + 6)^9$$

[7 points]

2. Find the derivative of the function $f(x) = \sin 2x(3x + 7)^5$.

[7 points]

3. Find $f'(5)$, if $f(x) = x^2 e^{0.2x}$

<div style="text-align: right">[8 points]</div>

4. Find an equation of the tangent line to the curve at the point $(0,0)$.

$$y = \sin x + \sin^2 x$$

<div style="text-align: right">[8 points]</div>

5. Find $f''(x)$, given the following

$$f(x) = x \cos x$$

[8 points]

6. Suppose that $f(2) = -3, g(2) = 2, f'(2) = -2,$ and $g'(2) = 1$. Find $h'(2)$

$$h(x) = [f(x)g(x)]^2$$

[8 points]

7. Two ships leave the same port at noon. Ship A sails north at 5 mph, and ship B sails east at 12 mph. How fast is the distance between them changing at 1 p.m.?

[8 points]

8. Find y' using implicit differentiaion

$$2y^2 - 3xy - x = 6$$

[8 points]

9. Find $\dfrac{dy}{dx}$, given $y = (\sin x)^{\cos x}$ (Hint: Use logarithmic differentiation).

[8 points]

10. Use differentials (or equivalently, a linear approximation) to approximate $\sqrt[3]{7.8}$.

[8 points]

11. Find the absolute maximum and minimum of the function on the closed interval $[-3, 1]$

$$f(x) = x^3 + 3x^2 - 1$$

[7 points]

12. The average speed of a vehicle on a stretch of Route 134 between 6 a.m. and 10 a.m. on a typical weekday is approximated by the function

$$f(t) = 20t - 40\sqrt{t} + 50, \qquad (0 \le t \le 4)$$

where $f(t)$ is measured in miles per hour and t is measured in hours, with $t = 0$ corresponding to 6 a.m. At what time of the morning commute is the traffic moving at the slowest rate? What is the average speed of a vehicle at that time?

[10 points]

13. Sketch the graph of $f(x) = x^2$ on the interval $0 \leq x \leq 2$, use your sketch to determine the absolute and relative maximum and minimum values of f.

[5 points]

Bonus. Use implicit differention to find the following derivative:

$$\frac{d}{dx}(\sec^{-1} x)$$

[6 points]