MATH 202 Exam II (6.1 - 7.3) Review

1. Know how to determine the area between curves.

2. Know how to find the volume of a solid using the Disk method and Shell Method.

3. Know how to determine the average value of a function.

4. Know how to determine work done.

5. Know how to use integration by parts.

6. Know how to use trigonometric substitution.

Example exercises: Quiz & Homework questions; Ch.6 Review, #7 - 16, 23, 25, 27, 28, 30; Ch.7 Review, #1 - 9, 11 - 20, 23, 24, 26 - 30;

A Few Worked Examples

1. Find the area of the region between the curves $f(x) = x^2+2x+1$, and $g(x) = 2x+5$.

First, we must find where $f(x)$ and $g(x)$ intersect. $f(x) = g(x) \Rightarrow x^2+2x+1 = 2x+5$ $\Rightarrow x^2 - 4 = 0 \Rightarrow x = \pm 2$. Now, $g(x) \geq f(x)$ on this interval so the area between the two curves is

$$A = \int_{-2}^{2} g(x) - f(x)\ dx = \int_{-2}^{2} 2x + 5 - (x^2 + 2x + 1)\ dx = \int_{-2}^{2} 4 - x^2$$

$$= 4x - \frac{1}{3}x^3\bigg|_{-2}^{2} = [4(2) - \frac{1}{3}(2^3)] - [4(-2) - \frac{1}{3}(-2)^3] = \frac{16}{3} - \left(-\frac{16}{3}\right) = \frac{32}{3}$$

2. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{25 - x^2}, y = 0, x = 2, x = 4$ about the x-axis.

The area of a cross section of the volume is $A(x) = \pi(\sqrt{25 - x^2})^2$.

$$\begin{aligned} V &= \int_2^4 A(x)\ dx = \int_2^4 \pi(\sqrt{25 - x^2})^2\ dx \\ &= \pi \textstyle\int_2^4 25 - x^2\ dx = \pi \left( 25x - \frac{1}{3}x^3 \bigg|_2^4 \right) \\ &= \pi([25(4) - \tfrac{1}{3}4^3] - [25(2) - \tfrac{1}{3}(2)^3]) = \pi\left(\frac{236}{3} - \frac{142}{3}\right) \\ &= \frac{94}{3}\pi \end{aligned}$$

3. Find the volume generated by rotating the region bounded by $y = x^2, y = 2 - x^2$, about $x = 1$.

The distance from the center of the rectangle to the axis of revolution is $p(x) = 1 - x$, and the height of the rectangle is $(2 - x^2) - x^2 = 2 - 2x^2$

$$V = \int_{-1}^{1} 2\pi(1-x)(2-2x^2)\ dx$$

$$= 2\pi \int_{-1}^{1} (1-x)(2-2x^2)\ dx = 2\pi \int_{-1}^{1} 2 - 2x - 2x^2 + 2x^3\ dx$$

$$= 4\pi \int_{-1}^{1} 1 - x - x^2 + x^3\ dx = 4\pi \left( x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \Big|_{-1}^{1} \right)$$

$$= 4\pi \left( \left[1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right] - \left[-1 - \frac{1}{2} - \frac{-1}{3} + \frac{1}{4}\right] \right)$$

$$= \frac{16\pi}{3}$$

4. Evaluate the following integral, $\displaystyle\int \frac{z}{10^z}\ dz$.

Let's try $u = z$ and $dv = \dfrac{1}{10^z}\ dz = 10^{-z}\ dz$, this means $du = dz$ and $v = -\dfrac{1}{\ln 10}(10^{-z})$

$$\int \frac{z}{10^z}\ dz = z\left(-\frac{1}{\ln 10}(10^{-z})\right) - \int -\frac{1}{\ln 10}(10^{-z})\ dz$$

$$= \left(-\frac{z}{\ln 10}(10^{-z})\right) - \frac{1}{(\ln 10)^2}(10^{-z}) + C$$

$$= -\frac{1}{\ln 10}(10^{-z})\left(z + \frac{1}{\ln 10}\right) + C$$