MATH 210 Sample Final SCORE (on next page)

Date____________________________ Name______________________________________

MATH 210
DISCRETE MATH
Sample FINAL

Show all work.

Do not begin until told to do so.

SCORE______________

1. Let $A = \{a, b, c, d, e\}, B = \{a, b, c, d, e, f, g, h\}, C = \{g, h, i\}$. Find the following:

   (a) $A \cup C$

   [2 points]

   (b) $A \cap C$

   [2 points]

   (c) $B - C$

   [2 points]

   (d) $B \cap C$

   [2 points]

   (e) $A \times C$

   [2 points]

2. Describe the sets in 1a and 1c from the previous problem using set builder notation.

   [5 points]

3. Use truth tables to show that $X \cap (X' \cup Y) = X \cup Y$.

[5 points]

4. Show that $X \cup (Y \cap Z) = (X \cup Y) \cap (Y \cup Z)$ using venn diagrams and element chasing.

[10 points]

5. Let $A = \{1, 2, 3\}$. Define the relation $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 1)\}$. Is R an equivalence relation? Explain why or why not.

[10 points]

6. Write the converse and contrapositive to the following statements:

(a) If $p(x)$ is a polynomial of odd degree, then $p(x)$ has at least one real root.

[2 points]

(b) If x is an even number, then $x^2 + 3x$ is an even number.

[2 points]

(c) Every Canadian is a Toronto Maple Leaf fan.

[2 points]

(d) If my dog is a dalmation, it has spots.

[2 points]

(e) If today is Thursday, then I have a test today.

[2 points]

7. Draw a binary tree for the following algebraic expression? $\dfrac{(x-2)^7 + (1+\frac{1}{x})^{1/2}}{9}$. Write the expression in Reverse Polish Notation (postfix form) and in Polish Notation (prefix form).

[10 points]

8. Prove by contradiction the following theorem "If $3n+2$ is odd, then n is odd." *(Hint: Assume $3n+2$ is odd, but n is even.)*

[6 points]

9. Use induction to prove that $1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$ for all positive integers n.

[10 points]

10. In a survey of the chewing gum preferences of baseball players, it was found that

- 22 like fruit.
- 25 like spearmint.
- 35 like grape.
- 9 like spearmint and fruit.
- 17 like fruit and grape.
- 20 like spearmint and grape.
- 6 like all flavors.
- 4 like none.

How many players were surveyed?

[6 points]

11. How many solutions to the equation $x + y + z = 12$ are there if x, y, and z must be non-negative numbers?

[6 points]

12. A man, woman, boy, girl, dog, and cat are walking down a long and winding road one after the other.

(a) In how many ways can this happen?

[2 points]

(b) In how many ways can this happen if the dog comes first?

[2 points]

(c) In how many ways can this happen if the dog immediately follows the boy?

[2 points]

(d) In how many ways can this happen if the dog (and only the dog) is between the man and the boy?

[2 points]

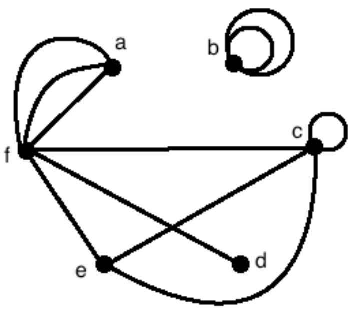
(e) In how many ways can this happen if the dog is between the man and the boy?

[2 points]

13. How many committees of five people can be chosen from 8 men and 11 women if exactly three women must be on each committee?

[5 points]

14. Find the adjacency matrix for the following graph.

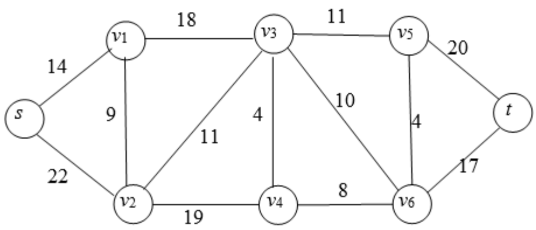


[5 points]

15. Find a graph whose adjacency matrix is $A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 4 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 \end{bmatrix}$.

[5 points]

16. Use Prim's Algorithm to find the minimum spanning tree for the following graph.



[5 points]

17. Use Dijkstra's Algorithm to find the shortest distance from the vertex s to each of the other vertices for the graph in problem 17.

[5 points]

18. Show that $[p \wedge q] \wedge [(\neg p) \vee (\neg q)]$ is a contradiction.

[5 points]

19. What values of the Boolean variables x and y satisfy $xy = x + y$?

[5 points]

20. Draw the Hasse diagram for the "greater than or equals" ($\geq$) relation on $\{0, 1, 2, 3, 4, 5\}$.

[5 points]

Bonus. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, with $n \geq 2$, given $a_0 = 6$ and $a_1 = 30$.

[12 points]