MATH 306 Exam I (1.1 - 1.7) Review

1. Know the definition of a linear system, solution to a linear system and how to solve a linear system

2. Know how to determine if a solution is exists and if a solution is unique.

3. Know how to recognize echelon forms, vector & matrix equations.

4. Know how to describe the solution sets of homogeneous and nonhomogeneous linear systems.

5. Know how to determine linear independence/dependence of a set.

Example exercises: Quiz & Homework questions; Ch.1 Supplementary Exercises, #1 - 19

A Few Worked Examples:

1. If $\boldsymbol{b} \neq \mathbf{0}$, can the solution set of $A\boldsymbol{x} = \boldsymbol{b}$ be a plane through the origin? Explain.

No. If the solution set of $A\boldsymbol{x} = \boldsymbol{b}$ contained the origin, then $\mathbf{0}$ would satisfy $A\mathbf{0} = \boldsymbol{b}$, which is not possible since $\boldsymbol{b} \neq \mathbf{0}$.

2. Under what conditions on b_1 and b_2 (if any) does $A\boldsymbol{x} = \boldsymbol{b}$ have a solution?

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Find two vectors which satisfy $A\boldsymbol{x} = \mathbf{0}$, and general solution to $A\boldsymbol{x} = \boldsymbol{b}$.

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 2 & 4 & 0 & 7 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & b_1 \\ 2 & 4 & 0 & 7 & b_2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 3 & b_1 \\ 0 & 0 & 0 & 1 & b_2 - 2b_1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & 7b_1 - 3b_2 \\ 0 & 0 & 0 & 1 & b_2 - 2b_1 \end{bmatrix}$$

No condition, there is a solution for any choice of b_1 and b_2; $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

both satisfy $A\boldsymbol{x} = \mathbf{0}$. The general solution is $\boldsymbol{x} = \begin{bmatrix} 7b_1 - 3b_2 \\ 0 \\ 0 \\ b_2 - 2b_1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

3. Suppose $\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}, \boldsymbol{v_4}$ are vecotrs in $\mathbb{R}^3$

   a) These four vectors are dependent because ______________.

   b) The two vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ will be dependent if ______________.

   c) The vectors $\boldsymbol{v_1}$ and $\mathbf{0}$ are dependent because ______________.

Solution:

a) The four vectors are columns of a 3 by 4 matrix with at least one free variable so $A\boldsymbol{x} = \mathbf{0}$ has a nontrivial solution.

b) The two vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ will be dependent if one is a multiple of the other.

c) The vectors $\boldsymbol{v_1}$ and $\mathbf{0}$ are dependent because $c_1\boldsymbol{v_1} + c_2\mathbf{0} = \mathbf{0}$ has a nontrivial solution (take any $c_2 \neq 0$).