MATH 306 Exam III (3.1 - 4.7) Review

1. Know the definition of determinant and how to find the determinant of a matrix.

2. Know the properties of the determinant and how to apply Cramer's rule to find the unique solution of the matrix equation $A\mathbf{x} = \mathbf{b}$.

3. Know the definition of vector spaces and subspaces and be able to determine if a given set with two operations defined is a vector space or subspace.

4. Know how to find a basis for the null space, row space, column space and left null space of a given matrix A.

5. Know how to determine if a set is linearly independent and if a set is a basis for a vector space.

6. Know how to find the dimension of a vector space.

7. Know the criteria for a transformation to be a linear transformation and what the kernel and range of a transformation are.

A Few Questions & Worked Examples

1. Determine the values of the parameter s for which the system has a unique solution, and describe the solution.

$$\begin{aligned} 6sx_1 + 4x_2 &= 5 \\ 9x_1 + 2sx_2 &= -2 \end{aligned}$$

2. Given the matrix

$$\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

Use row operations to show that det T $= (b-a)(c-a)(c-b)$.

$$\det \mathrm{T} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & (b-a)(b+a) \\ 0 & c-a & (c-a)(c+a) \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & (b+a) \\ 0 & 1 & (c+a) \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & (b+a) \\ 0 & 0 & (c-b) \end{vmatrix} = (b-a)(c-a)(c-b)$$

3. Suppose that $\mathbb{R}^4 = \text{Span}\,\{\boldsymbol{v_1}, \ldots, \boldsymbol{v_4}\}$. Explain why $\{\boldsymbol{v_1}, \ldots, \boldsymbol{v_4}\}$ is a basis for $\mathbb{R}^4$.

Let $A = [\boldsymbol{v_1}\ \boldsymbol{v_2}\ \boldsymbol{v_3}\ \boldsymbol{v_4}]$. Then A is square and its columns span $\mathbb{R}^4$ since $\mathbb{R}^4 = \text{Span}\,\{\boldsymbol{v_1}, \ldots, \boldsymbol{v_4}\}$ So its columns are linearly independent by the Invertible Matrix Theorem, and $\{\boldsymbol{v_1, v_2, v_3, v_4}\}$ is a basis for $\mathbb{R}^4$.

4. Use coordinate vectors to test the linear independence of the following set of polynomials.

$$(t-1)^2, t^3 - 2, (t-2)^3$$

Example exercises: Quiz & Homework questions;
Ch.3 Supplementary Exercises, #1 - 17;
Ch.4 Supplementary Exercises, #1 - 17.