1. Determine if the following vectors are linearly dependent or independent.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

[10 points]

2. Find the general solution of the homogeneous system below. Give your answer as a vector.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ 4x_1 + 7x_2 - 9x_3 &= 0 \\ -x_1 - 4x_2 + 9x_3 &= 0 \end{aligned}$$

[10 points]

3. Find the general solution to the following linear system:

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ -x_1 + x_2 - 7x_3 &= 8 \\ -2x_1 - 3x_2 + x_3 &= 1 \end{aligned}$$

Write solution in parametric vector form.

[10 points]

4. Constuct a 2×3 matrix A, not in echelon form, such that the solution of $A\boldsymbol{x} = \mathbf{0}$ is a plane in $\mathbb{R}^3$.

[5 points]

5. Compute the product or state that it is undefined.

a) $\begin{bmatrix} -2 & -2 & 6 \\ 5 & 8 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ -3 \\ 3 \end{bmatrix}$

[4 points]

b) $\begin{bmatrix} 5 & -3 \\ -3 & 4 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$

[4 points]

6. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -4 & 5 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

Determine if the equation $A\boldsymbol{x} = \boldsymbol{b}$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible b_1, b_2, b_3, give a description of the set of all $\boldsymbol{b}$ for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

[10 points]

7. Given $\boldsymbol{u} = \begin{bmatrix} -9 \\ 7 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$. Find the indicated vectors:

a) $\boldsymbol{u} + \boldsymbol{v}$

[3 points]

b) $\boldsymbol{v} - \boldsymbol{u}$

[3 points]

c) $5\boldsymbol{v}$

[3 points]

d) $-2\boldsymbol{u} + 5\boldsymbol{v}$

[3 points]

8. Let $\boldsymbol{a_1} = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$, $\boldsymbol{a_2} = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 2 \\ -10 \\ 6 \end{bmatrix}$

Determine whether $\boldsymbol{b}$ can be written as a linear combination of $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$. In other words, determine whether weights x_1 and x_2 exist, such that $x_1\boldsymbol{a_1} + x_2\boldsymbol{a_2} = \boldsymbol{b}$. If so, determine weights x_1 and x_2.

[10 points]