1. Compute the following determinant. (You can use a combination of row reduction and cofactor expansion.

$$\begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix}$$

[10 points]

2. Let A and P be square matrices, with P invertible. Show that $\det(PAP^{-1}) = \det(A)$.

[10 points]

3. Determine the values of the parameter s for which the system has a unique solution, and describe the solution. (Use Cramer's Rule)

$$\begin{array}{rcl}
6sx_1 + 4x_2 &=& 5\\
9x_1 + 2sx_2 &=& -2\\
\end{array} \\
[10 points]
\end{array}$$

4. Determine if the set of all polynomials in \mathbb{P}_n such that $\mathbf{p}(0) = 0$ is a subspace of \mathbb{P}_n (Remember that \mathbb{P}_n is the set of all polynomials of degree at most n).

[10 points]

5. Use coordinate vectors to test the linear independence of the following set of polynomials.

$$(t-1)^2, t^3-2, (t-2)^3$$

[10 points]

6. Let H be a n-dimensional subspace of an n-dimensional vector space V. Show that H = V.

[10 points]

7. Given the following matrix A, find bases for the Row A, and Nul A.

$$\left[\begin{array}{rrrrr} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{array}\right]$$

[10 points]

8. Mark each statement True or False. Justify each answer.

(a) If
$$A^3 = 0$$
, then det $A = 0$.
[3 points]

(b) Let V be a nonzero finite-dimensional vector space, and $S = \{v_1, \ldots, v_p\}$. If span S = V, then some subset of S is a basis for V.

[3 points]

(c) If a 3 by 3 matrix has det A = -1, then det 2A = -2.

[3 points]

(d) If A is $m \times n$ and the linear transformation $\boldsymbol{x} \mapsto A\boldsymbol{x}$ is onto, then rank A = m. [3 points]

(e) The nonzero rows of a matrix A form a basis for Row A.

[3 points]

9. Find the area of the parallelogram determined by the points (1,4), (-1,5), (3,9), and (5,8). How can you tell that the quadrilateral determined by the points is actually a parallelogram?

[10 points]

10. If the nullity of a 7×6 matrix A is 5, what are the dimensions of the column and row spaces of A?

[5 points]