SCORE_____

1. A 2-kg mass is attached to a spring with stiffness $k = 50$ N/m. The mass is displaced 1/4 m to the left of the equilibrium point and given a velocity of 1 m/sec to the left. Neglecting damping, find the equation of motion of the mass along with the amplitude, period and frequency. How long after release does the mass pass through the equilibrium position?

[12 points]

2. A 32-lb weight is attached to a vertical spring, causing it to stretch 6 in. upon coming to rest at equilibrium. The damping constant for the system is 2 lb-sec/ft. An external force $F(t) = 4\cos 8t$ lb is applied to the weight. Find the steady-state solution for the system.

[12 points]

3. Use the elimination method to find a general solution to the given linear system.

$$x' = x - y$$

$$y' = y - 4x$$

[12 points]

4. Solve the following system of differential equations.

$$x' = 3x - y - 12$$

$$y' = x + y + 4e^t$$

[12 points]

(Hint: Use determinants)

5. Solve the following differential equation.

$$y''' - 4y'' + y' + 6y = 0$$

[12 points]

6. Solve the differential equation

$$\frac{dy}{dx} + \frac{2y}{x} = 2x^2y^2$$

[12 points]

7. Solve the following system initial value problem.

$$y''' + y'' = \cos(2t), \;\; y(0) = 1, \;\; y'(0) = 2, \;\; y'(0) = 3$$

[12 points]

8. Solve the following differential equation using variation of parameters.

$$y''' + y'' = \sec t$$

[12 points]

9. Find a series solution for the following initial value problem:

$$y'' + xy' + y = 0,\ y(0) = 0,\ y'(0) = 1$$

[12 points]

10. Both $x = 0$ and $x = -1$ are singular points of the differential equation

$$x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0$$

Classify each as regular or irregular.

[12 points]

12. Solve the following Cauchy-Euler equation

$$x^2y'' + 2xy' - 4y = 0$$

[12 points]

13. Find a differential operator annihilates the given functions: a)$1 - 5x^2 + 8x^3$; b) e^{-3x}

[12 points]

14. Find a lower bound for the radius of convergence of the series solutions about $x = -1$ for the differential equation

$$(x^2 + 1)y'' + xy' - y = 0$$

[6 points]

Bonus. Find a series solution about the regular singular point $x = 0$ of

$$x^2y''(x) - xy'(x) + (1 - x)y(x) = 0,\ x > 0$$

using the method of Frobenius.

[20 points]