SCORE_____

1.A large tank is partially filled with 100 gallons of fluid in which 10 pounds of salt is dissolved. Brine containing $\frac{1}{2}$ pound of salt per gallon is pumped into the tank at a rate of 6 gal/min. The well-mixed solution is then pumped out at a slower rate of 4 gal/min. Find the number of pounds of salt in the tank after 30 minutes.

[15 points]

2. The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at any time. After 3 hours it is observed that there are 400 bacteria present. After 10 hours there are 2000 bacteria present. What was the initial number of bacteria? (Hint: First find the constant of proportionality, then you will be able to find the initial amount.)

[15 points]

3. A population model used in actuarial predictions is based on Gompertz equation. Solve Gompertz equation for P(t).

$$\frac{dP}{dt} = P(a - b \ln P)$$

where a and b are constants. (Hint: Use substitution to transform the equation to a linear DE).

[10 points]

4. Solve the following differential equation

$$(9x^2 - y^2)\ dx + (xy - 3x^3y^{-1})\ dy = 0$$

[10 points]

5. Use the method for solving Bernoulli equations to solve the following differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = 4x^2y^2$$

[10 points]

6. Solve the equation

$$(2xy^{-1})\ dx + (2 - 4x^2y^{-2})\ dy = 0$$

[10 points]

7. Use the method for solving equations with linear coefficients to solve the following differential equation.

$$(2x - y)\ dx + (4x + y - 3)\ dy = 0$$

[10 points]

8. Consider the initial value problem given below.

$$y' = x + 4\cos(xy), \quad y(0) = 0$$

Use the improved Euler's method subroutine with step size $h = 0.4$ to approximate the initial value problem at points $x = 0.0, 0.4, 0.8$.

[10 points]