1. Solve the following differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 3t^2 - 5 + e^t$$

[10 points]

2. Solve the following differential equation by variation of parameters.

$$y'' + y = \sin t$$

[10 points]

3. Find a particular solution (method of undetermined coefficients) of
$$y'' - 2y' + y = e^x$$

[10 points]

4. Rewrite $\cos(2x) - 2\sin(2x)$ in the form $A\cos(2x + \phi)$.

[10 points]

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5. A 3-kg mass is attached to a spring with stiffness $k = 48N/m$. If the mass is pushed up 1/2 m and released with a velocity of 2m/sec down, find the amplitude, period, and frequency of the resulting motion, neglecting air resistance.

[10 points]

6. Solve the following system of differential equations by either elimination or determinants.

$$\frac{dy}{dt} = x$$

$$\frac{dx}{dt} = 2x - y$$

[10 points]

7. A nonhomogeneous equation and a particular solution are given. Find a general solution for the equation.

$$y'' + 12y' + 32y = 128x^2 + 96x + 8 + 45e^x, \quad y_p(x) = e^x + 4x^2$$

[10 points]

8. Solve the following differential equation

$$y''' - y'' - 18y' - 10y = 0$$

[12 points]

4

9. Solve the initial value problem below for the Cauchy-Euler equation.

$$t^2 y''(t) - 5ty'(t) + 5y(t) = 0, \ y(1) = 4, \ y'(1) = -4$$

[12 points]